

Trading in Risk Dimensions (TRD)
Lester Ingber

Lester Ingber Research (LIR)

ingber@ingber.com

<http://www.ingber.com/> • <http://alumnus.caltech.edu/~ingber>

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Abstract

Previous work, mostly published, developed two-shell recursive trading systems. An inner-shell of Canonical Momenta Indicators (CMI) is adaptively fit to incoming market data. A parameterized trading-rule outer-shell uses the global optimization code Adaptive Simulated Annealing (ASA) to fit the trading system to historical data. A simple fitting algorithm, usually not requiring ASA, is used for the inner-shell fit. An additional risk-management middle-shell has been added to create a three-shell recursive optimization/sampling/fitting algorithm. Portfolio-level distributions of copula-transformed multivariate distributions (with constituent markets possessing different marginal distributions in returns space) are generated by Monte Carlo samplings. ASA is used to importance-sample weightings of these markets.

The core code, Trading in Risk Dimensions (TRD), processes Training and Testing trading systems on historical data, and consistently interacts with RealTime trading platforms at minute resolutions, but this scale can be modified. This approach transforms constituent probability distributions into a common space where it makes sense to develop correlations to further develop probability distributions and risk/uncertainty analyses of the full portfolio. ASA is used for importance-sampling these distributions and for optimizing system parameters.

1. Background

This work is largely based on previous work in several disciplines using a similar formulation of multivariate nonlinear nonequilibrium systems [13-15], using powerful numerical algorithms to fit models to data [12]. A published report closest to this project was formulated for a portfolio of options [16].

1.1. Adaptive Simulated Annealing (ASA)

Adaptive Simulated Annealing (ASA) [10] is used in TRD to optimize trading-rule parameters and to importance-sample contracts for risk-management.

ASA is a C-language code developed to statistically find the best global fit of a nonlinear constrained non-convex cost-function over a D -dimensional space. This algorithm permits an annealing schedule for “temperature” T decreasing exponentially in annealing-time k , $T = T_0 \exp(-ck^{1/D})$. The introduction of re-annealing also permits adaptation to changing sensitivities in the multi-dimensional parameter-space. This annealing schedule is faster than fast Cauchy annealing, where $T = T_0/k$, and much faster than Boltzmann annealing, where $T = T_0/\ln k$. ASA has over 100 OPTIONS to provide robust tuning over many classes of nonlinear stochastic systems.

1.2. Generic Approach

The approach to risk management presented here is quite generic. That is, while the tools used in this project can be used in other projects, each project requires its own extensive research to establish appropriate cost functions to optimize or importance-sample, and each project requires its own intensive research to efficiently tune ASA. So, while there are many projects that might productively use these algorithms, each system requires its own extensive and intensive R&D.

For example, other current projects include developing tools for decision-makers of companies and government agencies, assembling functions, departments, processes, etc., tailored to specific client requirements, into an overall “portfolio” from which top-level measures of performance are developed with associated measures of risk, and with audit trails back to the member constituents. Another project of interest is developing “portfolios” of physiological indicators, e.g., to merge different imaging data to enhance clinical diagnoses [18]. A project which directly uses some of the TRD algorithms is Ideas by Statistical Mechanics (ISM) [17]. ISM models evolution and propagation of ideas/patterns throughout populations subjected to endogenous and exogenous interactions. The project Real Options for Project Schedules (ROPS) uses TRD algorithms for risk analysis among generic projects [19].

2. Data

A time epoch is chosen to measure possible trading intervals, WINDOW_e .

Enough price (and volume, etc.) data, e.g., hundreds of epochs, is used to gather some representative statistics including “outliers.” Define

$$dx = \frac{p_t - p_{t-1}}{p_{t-1}} \equiv \frac{p_t - p_{t'}}{p_{t'}} \quad (1)$$

Market differenced-variables $\{dx\}$ lead to portfolio variables dM . Here, dM signifies the portfolio returns, based on portfolio values $\{K_i\}$, e.g.,

$$dM_t = \frac{K_t - K_{t-1}}{K_{t-1}} \quad (2)$$

3. Exponential Marginal Distribution Models

This paper uses exponential distributions to detail the TRD algorithms. However, the TRD code has hooks to work with other distributions.

Assume that each market is fit well to a two-tailed exponential density distribution p (not to be confused with the indexed price variable p_t) with scale χ and mean m ,

$$p(dx)dx = \begin{cases} \frac{1}{2\chi} e^{-\frac{dx-m}{\chi}} dx, & dx \geq m \\ \frac{1}{2\chi} e^{\frac{dx-m}{\chi}} dx, & dx < m \end{cases}$$

$$= \frac{1}{2\chi} e^{-\frac{|dx-m|}{\chi}} dx \quad (3)$$

which has a cumulative probability distribution

$$F(dx) = \int_{-\infty}^{dx} dx' p(dx') = \frac{1}{2} \left[1 + \operatorname{sgn}(dx - m) \left(1 - e^{-\frac{|dx-m|}{\chi}} \right) \right] \quad (4)$$

where χ and m are defined by averages $\langle . \rangle$ over a window of data,

$$m = \langle dx \rangle$$

$$2\chi^2 = \langle (dx)^2 \rangle - \langle dx \rangle^2 \quad (5)$$

The $p(dx)$ are “marginal” distributions observed in the market, modeled to fit the above algebraic form. Note that the exponential distribution has an infinite number of non-zero cumulants, so that $\langle dx^2 \rangle - \langle dx \rangle^2$ does not have the same “variance” meaning for this “width” as it does for a Gaussian distribution which has just two independent cumulants (and all cumulants greater than the second vanish). Below algorithms are specified to address correlated markets giving rise to the stochastic behavior of these markets.

The TRD code can be easily modified to utilize distributions $p'(dx)$ with different widths, e.g., different χ' for dx less than and greater than m ,

$$p'(dx)dx = \frac{1}{2\chi'} e^{-\frac{|dx-m|}{\chi'}} dx \quad (6)$$

3.1. Fitting Data

Note that to establish the exponential distribution, (current-time sensitive exponential) moving averages are used to force data into a specific functional form, a form which must be regularly checked for its statistical significance.

4. Copula Transformation

4.1. Aside on Use of Gaussian Copula

Gaussian copulas are developed in TRD. Other copula distributions are possible, e.g., Student-t distributions (often touted as being more sensitive to fat-tailed distributions — here data is first adaptively fit to fat-tailed distributions prior to copula transformations). These alternative distributions can be quite slow because inverse transformations typically are not as quick as for the present distribution.

Copulas are cited as an important component of risk management not yet widely used by risk management practitioners [1]. Gaussian copulas are presently regarded as the Basel II standard for credit risk management [7]. While real-time risk-management for intra-day trading is becoming more popular, most approaches still use simpler VaR measures [3]. TRD permits fast as well as robust copula risk management in real time.

The copula approach can be extended to more general distributions than those considered here [9]. If there are not analytic or relatively standard math functions for the transformations (and/or inverse transformations described) here, then these transformations must be performed explicitly numerically in code such as TRD. Then, the ASA_PARALLEL OPTIONS already existing in ASA (developed as part of the 1994 National Science Foundation Parallelizing ASA and PATHINT Project (PAPP)) would be very useful to speed up real time calculations [10].

4.2. Transformation to Gaussian Marginal Distributions

A Normal Gaussian distribution has the form

$$p(dy) = \frac{1}{\sqrt{2\pi}} e^{-\frac{dy^2}{2}} \quad (7)$$

with a cumulative distribution

$$F(dy) = \frac{1}{2} \left[1 + \operatorname{erf}\left(\frac{dy}{\sqrt{2}}\right) \right] \quad (8)$$

where the erf() function is a tabulated function,

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x dx' e^{-x'^2} \quad (9)$$

By setting the numerical values of the above two cumulative distributions, monotonic on interval [0,1], equal to each other, the transformation of the x marginal variables to the y marginal variables is effected,

$$\begin{aligned} dy &= \sqrt{2} \operatorname{erf}^{-1}(2F(dx) - 1) \\ &= \sqrt{2} \operatorname{sgn}(dx - m) \operatorname{erf}^{-1}\left(1 - e^{-\frac{|dx-m|}{\chi}}\right) \end{aligned} \quad (10)$$

The inverse mapping is used when applying this to the portfolio distribution. Note that

$$dy \geq 0 \rightarrow dx - m \geq 0$$

$$dy < 0 \rightarrow dx - m < 0$$

$$\operatorname{erf}([\cdot]) = -\operatorname{erf}(-[\cdot]) \quad (11)$$

yields

$$dx = m - \text{sgn}(dy) \chi \ln \left[1 - \text{erf} \left(\frac{|dy|}{\sqrt{2}} \right) \right] \quad (12)$$

4.3. Including Correlations

To understand how correlations enter, look at the stochastic process defined by the dy^i marginal transformed variables:

$$dy^i = \hat{g}^i dw_i \quad (13)$$

where dw_i is the Wiener Gaussian noise contributing to dy^i of market i . The transformations are chosen such that $\hat{g}^i = 1$.

Now, a given market's noise, ($\hat{g}^i dw_i$), has potential contributions from all N markets, which is modeled in terms of N independent Gaussian processes, dz_k ,

$$\hat{g}^i dw_i = \sum_k \hat{g}_k^i dz_k \quad (14)$$

The covariance matrix (g^{ij}) of these y variables is then given by

$$g^{ij} = \sum_k \hat{g}_k^i \hat{g}_k^j \quad (15)$$

with inverse matrix, the “metric,” written as (g_{ij}) and determinant of (g^{ij}) written as g .

Since Gaussian variables are now being used, the covariance matrix is calculated directly from the transformed data using standard statistics, the point of this “copula” transformation [26,28].

Correlations ρ^{ij} are derived from bilinear combinations of market volatilities

$$\rho^{ij} = \frac{g^{ij}}{\sqrt{g^{ii} g^{jj}}} \quad (16)$$

Since the transformation to Gaussian space has defined $g^{ii} = 1$, here the covariance matrices theoretically are identical to the correlation matrices.

This transformation is rigorously enforced. I.e., a finite sample of Gaussian-transformed (prefiltered) returns data will not yield a covariance matrix equal to its correlation matrix, so the covariance matrix is properly normalized (by dividing by the sqrt of the products of the diagonal elements). This step affords some statistical robustness of this procedure over moving windows of data.

This gives a multivariate correlated process P in the dy variables, in terms of Lagrangians L and Actions A ,

$$P(dy) \equiv P(dy^1, \dots, dy^N) = (2\pi dt)^{-\frac{N}{2}} g^{-\frac{1}{2}} e^{-Ldt} \quad (17)$$

where $dt = 1$ above. The Lagrangian L is given by

$$L = \frac{1}{2dt^2} \sum_{ij} dy^i g_{ij} dy^j \quad (18)$$

The effective action A_{eff} , presenting a “cost function” useful for sampling and optimization, is defined by

$$P(dy) = e^{-A_{\text{eff}}}$$

$$A_{\text{eff}} = Ldt + \frac{1}{2} \ln g + \frac{N}{2} \ln(2\pi dt) \quad (19)$$

4.3.1. Stable Covariance Matrices

Covariance matrices, and their inverses (metric), are known to be quite noisy, so often they must be further developed/filtered for proper risk management. The root cause of this noise is recognized as “volatility of volatility” present in market dynamics, which essentially doubles the number of stochastic

variables to consider [23]. In addition to such problems, ill-conditioned matrices can arise from loss of precision for large variables sets, e.g., when calculating inverse matrices and determinants as required here. In general, the window size used for covariance calculations should exceed the number of market variables to help tame such problems.

A very good approach for avoiding ill-conditioning and lack of positive-definite matrices is to perform pre-averaging of input data using a window of three epochs [25]. Other methods in the literature include subtracting eigenvalues of parameterized random matrices [24]. Using Gaussian transformed data alleviates problems usually encountered with fat-tailed distributions. Selection of reasonable windows, coupled with pre-averaging, seems to robustly avoid ill-conditioning. This was tested in portfolios of 20-100 markets.

The covariance matrix of the historical [0, 1] Gaussian transformed exponentially distributed returns is to be used for forecasts of future returns. As mentioned in the previous section, after calculating the covariance matrix from an historical sample, it is renormalized to StdDevs of 1, i.e., defining the correlation matrix consistent with the theoretical transformation above.

The combination of moving-average windows, 3-epochs pre-averaging, and renormalization effectively gets rid of negative eigenvalues over all epochs of data.

4.4. Fitting Data

Results must be tested with respect to WINDOW _{χ} sizes for updated covariance calculations, and with respect to using standard versus exponential moving averages. Tests should be performed regularly for ill-conditioned matrices.

4.5. Copula of Multivariate Correlated Distribution

The multivariate distribution in x -space is specified, including correlations, using

$$P(dx) = P(dy) \left| \frac{\partial dy^i}{\partial dx^j} \right| \quad (20)$$

where $\left| \frac{\partial dy^i}{\partial dx^j} \right|$ is the Jacobian matrix specifying this transformation. This gives

$$P(dx) = g^{-\frac{1}{2}} e^{-\frac{1}{2} \sum_{ij} (dy_{dx}^i)^\dagger (g_{ij} - I_{ij}) (dy_{dx}^j)} \prod_i P_i(dx^i) \quad (21)$$

where (dy_{dx}^i) is the column-vector of $(dy_{dx}^1, \dots, dy_{dx}^N)$ expressed back in terms of their respective (dx^1, \dots, dx^N) , $(dy_{dx}^i)^\dagger$ is the transpose row-vector, and (I) is the identity matrix (all ones on the diagonal).

The Gaussian copula $C(dx)$ is defined from Eq. (21),

$$C(dx) = g^{-\frac{1}{2}} e^{-\frac{1}{2} \sum_{ij} (dy_{dx}^i)^\dagger (g_{ij} - I_{ij}) (dy_{dx}^j)} \quad (22)$$

5. Portfolio Distribution

The probability density $P(dM)$ of portfolio returns dM is given as

$$P(dM) = \int \prod_i d(dx^i) P(dx) \delta_D(dM_t - \sum_j (a_{j,t} dx^j + b_{j,t})) \quad (23)$$

where the Dirac delta-function δ_D expresses the constraint that

$$dM = \sum_j (a_j dx^j + b_j) \quad (24)$$

The coefficients a_j and b_j are determined by specification of the portfolio current K_t , and “forecasted” K_t , giving the returns expected at t , dM_t ,

$$dM_t = \frac{K_t - K_{t'}}{K_{t'}}$$

$$K_{t'} = Y_{t'} + \sum_i \text{sgn}(NC_{i,t'}) NC_{i,t'} (p_{i,t'} - p_{i,@,t'})$$

$$K_t = Y_t + \sum_i (\text{sgn}(NC_{i,t}) NC_{i,t} (p_{i,t} - p_{i,@,t}) + SL[NC_{i,t} - NC_{i,t'}]) \quad (25)$$

where $NC_{i,t}$ is the current number of broker-filled contracts of market i at time t ($NC > 0$ for long and $NC < 0$ for short positions), $p_{i,@,t'}$ and $p_{i,@,t}$ are the long/short prices at which contracts were bought/sold according to the long/short signal $\text{sgn}(NC_{i,t'})$ generated by external models. For simulations of models that trade at the open, the previous close is taken as a proxy for these entry prices. Note that all prices are in dollars at this point of the calculation, e.g., including any required FX transformations. Y_t and $Y_{t'}$ are the dollars available for investment. The function SL is the slippage and commissions suffered by changing the number of contracts. The prices $p_{i,t}$ are developed in terms of the price returns, Eq. (1), and the prices $p_{i,t'}$.

If trading signals were not generated in this project, a_j do not depend on the prices $p_{i,t}$. I.e., trading rules dependent on $p_{i,t}$ cannot cause changes from long (short) to short (long) positions, since trading signals are not generated for forecast prices $p_{i,t}$ (which TRD can be set to process, as discussed below). However, changes in $\text{sgn}(NC_{i,t'})$ still can result from the sampling process of NC 's.

The function $\text{sgn}(NC_{i,t})$ is be a multivariate function if trading rules depend on correlated markets. Furthermore, a proper trading system distinguishes bid and ask prices, which are taken into account in the trading functions $\text{sgn}(NC_{i,t})$ and $\text{sgn}(NC_{i,t'})$ and the cost of trading/cash modifications that enter into the calculation of portfolio returns.

If futures are traded there are no appreciating“asset” values to buying or selling contracts; only changes in positions would matter. The profit/loss is calculated as

$$PL_t = K_t - K_{t'} \quad (26)$$

The time scale is set by the data. If $\text{WINDOW}_e = dt = 1$ epoch, then the $w_{i,t}$ are forecast for the next epoch. If $\text{WINDOW}_e = dt = 20$ epochs, then the $w_{i,t}$ are projected for 20 epochs, etc. It makes good sense to develop complementary risk management procedures at multiple time scales for all input data, calculations, etc.

5.1. Recursive Risk-Management in Trading Systems

Sensible development of trading systems fit trading-rule parameters to generate the “best” portfolio (best depends on the chosen criteria).

This necessitates fitting risk-managed contract sizes to chosen risk targets, for each set of chosen trading-rule parameters, e.g., selected by an optimization algorithm. A given set of trading-rule parameters affects the $a_{j,t}$ and $b_{j,t}$ coefficients in Eq. (23) as these rules act on the forecasted market prices as they are generated to sample the multivariate market distributions.

This process must be repeated as the trading-rule parameter space is sampled to fit the trading cost function, e.g., based on profit, Sharpe ratio, etc., of the Portfolio returns.

6. Risk Management

Once $P(dM)$ is developed (e.g., numerically), risk-management optimization is defined. The portfolio integral constraint is,

$$Q = P(dM < VaR) = \int_{-\infty}^{-|VaR|} dM P(M_t | M'_{t'}) \quad (27)$$

where VaR is a fixed percentage of the total available money to invest. E.g., this is specifically implemented as

$$VaR = 0.05, Q = 0.01 \quad (28)$$

where the value of VaR is understood to represent a possible 5% loss in portfolio returns in one epoch, e.g., which approximately translates into a 1% chance of a 20% loss within 20 epochs. Expected tail loss (ETL), sometimes called conditional VaR or worst conditional expectation, can be directly calculated as an average over the tail. While the VaR is useful to determine expected loss if a tail event does not occur, ETL is useful to determine what can be lost if a tail event occurs [4].

ASA [10] is used to sample future contracts defined by a cost function, e.g., maximum profit, subject to the constraint

$$Cost_Q = |Q - 0.01| \quad (29)$$

by optimizing the $NC_{i,t}$ parameters. Other post-sampling constraints described below can then be applied. (Judgments always must be made whether to apply specific constraints, before, during or after sampling.)

Note that this definition of risk does not take into account accumulated losses over folding of the basic time interval used to define $P(dM)$, nor does it address any maximal loss that might be incurred within this interval (unless this is explicitly added as another constraint function).

Risk management is developed by (ASA-)sampling the space of the next epoch's $\{NC_{i,t}\}$ to fit the above Q constraint using the sampled market variables $\{dx\}$. The combinatoric space of NC 's satisfying the Q constraint is huge, and so additional NC -models are used to choose the actual traded $\{NC_{i,t}\}$.

7. Sampling Multivariate Normal Distribution for Events

Eq. (23) certainly is the core equation, the basic foundation, of most work in risk management of portfolios. For general probabilities not Gaussian, and when including correlations, this equation cannot be solved analytically.

Other people approximate/mutilate this multiple integral to attempt to get some analytic expression. Their results may in some cases serve as interesting "toy" models to study some extreme cases of variables, but there is no reasonable way to estimate how much of the core calculation has been destroyed in this process.

Many people resort to Monte Carlo sampling of this multiple integral. ASA has an ASA_SAMPLE option that similarly could be applied. However, there are published algorithms specifically for multivariate Normal distributions [6].

7.1. Transformation to Independent Variables

The multivariate correlated dy variables are further transformed into independent uncorrelated Gaussian dz variables. Multiple Normal random numbers are generated for each dz^i variable, subsequently transforming back to dy , dx , and dp variables to enforce the Dirac δ -function constraint specifying the VaR constraint.

The method of Cholesky decomposition is used (eigenvalue decomposition also could be used, requiring inverses of matrices, which are used elsewhere in this project), wherein the covariance matrix is factored into a product of triangular matrices, simply related to each other by the adjoint operation. This is possible because G is a symmetric positive-definite matrix, i.e, because care has been taken to process the raw data to preserve this structure as discussed previously.

$$G = (g^{ij}) = C^\dagger C$$

$$I = C G^{-1} C^\dagger \quad (30)$$

from which the transformation of the dy to dz are obtained. Each dz has 0 mean and StdDev 1, so its covariance matrix is 1:

$$\begin{aligned} I &= \langle (dz)^\dagger (dz) \rangle \\ &= \langle (dz)^\dagger (C G^{-1} C^\dagger) (dz) \rangle \end{aligned}$$

$$\begin{aligned}
&= \langle (C^\dagger dz)^\dagger G^{-1} (C^\dagger dz) \rangle \\
&= \langle (dy)^\dagger G^{-1} (dy) \rangle
\end{aligned} \tag{31}$$

where

$$dy = C^\dagger dz \tag{32}$$

The collection of related $\{dx\}$, $\{dy\}$, and $\{dz\}$ sampled points are defined here as Events related to market movements.

8. Numerical Development of Portfolio Returns

8.1. X From Sampled Events Into Bins

One approach is to directly develop the portfolio-returns distribution, from which moments are calculated to define Q . This approach has the virtue of explicitly exhibiting the portfolio distribution being used.

The sampling process of Events are used to generate portfolio-return Bins to determine the shape of $P(dM)$. Based on prior analyses of data — market distributions have been assumed to be basically two-tailed exponentials — here too prior analyses strongly supports two-tailed distributions for the portfolio returns. Therefore, only a “reasonable” sampling of points of the portfolio distribution, expressed as Bins, is needed to calculate the moments. For example, a base function to be fitted to the Bins would be in terms of parameters, width X and mean m_M ,

$$\begin{aligned}
P(dM)dM &= \begin{cases} \frac{1}{2X} e^{-\frac{dM-m_M}{X}} dM, & dM \geq m_M \\ \frac{1}{2X} e^{\frac{dM-m_M}{X}} dM, & dM < m_M \end{cases} \\
&= \frac{1}{2X} e^{-\frac{|dM-m_M|}{X}} dM
\end{aligned} \tag{33}$$

X and m_M are defined from data in the Bins by

$$\begin{aligned}
m_M &= \langle dM \rangle \\
2X^2 &= \langle (dM)^2 \rangle - \langle dM \rangle^2
\end{aligned} \tag{34}$$

By virtue of the sampling construction of $P(dM)$, X implicitly contains all correlation information inherent in A'_{eff} .

The TRD code can be easily modified to utilize distributions $P'(dM)$ with different widths, e.g., different X' for dM less than and greater than m_M ,

$$P'(dM)dM = \frac{1}{2X'} e^{-\frac{|dM-m_M|}{X'}} dM \tag{35}$$

A large number of Events populate Bins into the tails of $P(dM)$. Different regions of $P(dM)$ could be used to calculate a piecewise X to compare to one X over the full region, with respect to sensitivities of values obtained for Q ,

$$Q = \frac{1}{2} e^{-\frac{|VaR-m_M|}{X}} \tag{37}$$

Note that fixing Q , VaR , and m_M fixes the full shape of the portfolio exponential distribution. Sampling of the NC_i is used to adapt to this shape constraint.

8.1.1. Events Without Bins

Some experience with estimating moments of Events demonstrates that it is not always necessary to coarse-grain the sampling of these Events. I.e., Bins need not be used in these cases, and the population of Events are directly use to estimate the moments of the portfolio distribution.

8.2. X Directly From Moments of Events

8.2.1. Moments With No Trading Functions

In production runs, the shape of the portfolio-returns distribution is not essential, as only the first two moments are required to determine Q . This approach increases the efficiency of the ASA optimization over NC -space by orders of magnitude since the Events are used prior to entering ASA.

Eq. (24) is used to decouple the calculation of moments over the Events, requiring only market distributions of the next epoch's market returns, from the next NC factors which are optimized by ASA. The moments of these market distributions are used within the ASA optimization to determine multiple sets of NC 's, to yield moments of the portfolio-return distribution.

Expectations are taken over the copula-enhanced multivariate distribution. The first moments are calculated by

$$\begin{aligned} \langle dM \rangle &= \langle \sum_j (a_j dx^j + b_j) \rangle \\ &= \sum_j (a_j dm_j + b_j) \end{aligned} \quad (38)$$

since expectations over one market is the same as taking the expectation over the marginal distributions. The second cumulants are calculated by

$$\langle (dM - \langle dM \rangle)^2 \rangle = \langle (\sum_j a_j (dx^j - m_j))^2 \rangle \quad (39)$$

which requires calculating the "width" matrix

$$h^{ij} = \langle dx^i dx^j \rangle - \langle dx^i \rangle \langle dx^j \rangle \quad (40)$$

The term "covariance" is reserved for the 2nd cumulant (2nd moment minus square of 1st moment) of a Gaussian distribution, the rationale for applying the copula approach above.

8.2.1.1. Variate Corrections

Some additional numerical precision in calculating moments is attained by using a variate numerical approach, used effectively in options calculations [8]. This procedure cancels a lot of systematic error introduced by meshes and sampling biases.

$$\langle . \rangle_v = \langle . \rangle_p - \langle . \rangle_e + E \quad (41)$$

$\langle . \rangle_v$ is the variate result to be used in the project. $\langle . \rangle_p$ is the numerical (sampling via Genz -> Cholesky -> Gaussian -> Exponential) expectation with respect to a given distribution (here the copula enhanced multivariate exponential distribution). $\langle . \rangle_e$ is the numerical expectation, using the same sampling points as used for $\langle . \rangle_p$, but using a simple distribution close to the true distribution, for which an exact closed-form solution E is known. For example, if $\langle . \rangle_e$ is taken over the exponential marginal distributions, then exact widths are calculated by

$$E_{ij} = g^{ij} = \frac{1}{2} \delta_{ij} \chi^2 \quad (42)$$

where δ_{ij} is the Kronecker delta function, equal to 1 if $i = j$ and 0 otherwise.

In many instances, the correlations test the mesh of Events more than the shape of the exponential distributions. The variate correction used numerically subtracts and analytically adds moments of both multivariate exponential and Gaussian distributions, the latter using the calculated correlation matrix.

8.2.2. Moments With Trading Functions

If the signal strengths of all markets are to be kept constant for the next epoch while risk-management of contract sizes is being performed, then the algorithm in Section [8.2.1.] is most efficient.

As remarked below Eq. (25), if trading rules are being fit to select trading signals, then the a_j and b_j coefficients are not the same for each sampling of the multivariate market distributions. If these effects are to be considered, then the recursive method described in Section [5.1.] must be applied using the algorithm in Section [8.1.].

Variate corrections are used as for the No Trading calculations, but they are not as corrective since trading rules incur nonlinear dependencies on postpoint market prices.

This approach averages over all signals generated from all Events to determine an average signal to be used in the calculation of the moments of the portfolio distribution. Note that the algorithm used does not alter the calculation of prepoint covariance matrices, using pre-filtered data as discussed previously, since the new Events are not used to update these.

9. NC Models

Optimization of Q within any level of precision does not at all return a unique set of orders NC 's to place at the next epoch.

Within the resolution used for $NC_{i,t}$, multiple sets of $NC_{i,t}^m$, labeled by m , are obtained by turning on MULTI_MIN in ASA [10]. This circumstance must be exploited to select the next NC 's. ASA samples such a large space constrained by relatively few equations and is limited to a simple one-number cost function (which may include multiple constraints).

A Lagrange multiplier term is added to the primary cost function to constrain scaling; i.e., a simple multiple scaling of all contracts can effect the Q risk constraint. The actual Lagrange multiplier is considered to be absorbed into the constraint parameters.

In general, multiple minima close to the global minimum, defined by the selected parameter resolution, are approximately equally good candidate solutions. There is some tradeoff in having prefilter constraints within the ASA optimization, versus having (a sequence of) postfilter NC models applied after the NC optimization.

9.1. Individual-Market VaR_i Prefilter

One good prefilter is to set individual market constraints that can give reasonable bounds to NC space prior to entering ASA.

Similar to the portfolio constraint, levels of risk, q_i , must be specified to lose a given level of each component of the portfolio, VaR_i , e.g., 5%. Once q_i is specified, e.g., $q_i = 0.01$, the maximum long or minimum short level for a market is specified on any given epoch for a given number of contracts.

The individual market constraints $\{q_i\}$ are calculated straightforwardly using the Events.

Each epoch, prior to entering ASA, a set of constraints on NC_i is set to specify max and min ranges of the differenced NC_i ASA parameters, e.g., by solving for $NC_{i,max}$ ($NC_{i,min}$ for a short),

$$VaR_i = \frac{NC_{i,t,max}(p_{i,t,tail} - p_{i,@,t}) + SL(NC_{i,t,max} - NC_{i,t-1}) - NC_{i,t-1}(p_{i,t-1} - p_{i,@,t-1})}{\sum_j [NC_{j,t-1}(p_{j,t-1} - p_{j,@,t-1})]} \quad (43)$$

where $p_{i,t,tail}$ is either the edge of the 1% tail, $p_{i,t,1p}$, or an ensemble average over this loss region, $\langle p_{i,t} \rangle_L$. Since $|p_{i,t,1p}| < |\langle p_{i,t} \rangle_L|$, the use of $\langle p_{i,t} \rangle_L$ gives smaller, more conservative, values of $NC_{i,t,max}$. The use of $\langle p_{i,t} \rangle_L$ is more in line with current research on "conditional VaR" [27]. (However, in line with another risk constraint, the edge of the 1% region is used to calculate the portfolio risk Q .)

The entry prices, $p_{i,@,t}$, for the next epoch are taken to be those calculated at current closes. Note that the threshold prices $p_{i,t,tail}$ are backed out of the one-percentile returns in the tails of the projected/component individual-market distributions, as they enter in the basic Eq. (25).

The strengths of the signals, s_i , weights determined by averaging over multiple trading models, are also used to obtain resultant ASA ranges for the differenced NC_i parameters, [min, max] (for a long position) such that the next orders are constrained as

$$\begin{aligned} \min_i &= \text{MIN} \left[1, \frac{|s_i|}{\langle |s_j| \rangle} \right] \\ \max_i &= \text{MAX} \left[\min_i, \text{MIN} \left[NC_{i,t-1,max}, NC_{i,t-1,max} \frac{|s_i|}{\langle |s_j| \rangle} \right] \right] \end{aligned} \quad (44)$$

where MAX and MIN are functions defining maximum and minimum elements of their arguments. There are similar expressions for short positions.

9.2. Candidate Postfilters

Multivariate postfilters are added according to a user-defined weight. The resultant set of “best” contracts is the weighted average of all user-defined postfilters.

9.2.1. Diversification of dNC Including Correlations

For example, the orders $dNC_{i,t} = NC_{i,t} - NC_{i,t'}$ to be used the next open can be selected from ASA’s MULTI_MIN set by weighting according to

$$\text{MIN} \sum_{i,j} dNC_i \rho_{ij} dNC_j \quad (45)$$

using the inverse-correlation matrix ρ_{ij} developed using the copula approach.

This approach tends to keep trading activity dispersed among the markets. In the limits of high positive or negative correlations, weights of these correlated market contributions collapse properly to smaller weights.

9.2.2. Diversification of dNC Including Covariances

In Eq. (45), ρ_{ij} can be replaced by the full metric, the inverse width matrix, h_{ij} , calculated from the moments of the next epoch’s multivariate market distribution. This model “risk-weights” each contribution by an inverse correlated “volatility” as it enters naturally in the width matrix.

9.2.3. Maximization of P/L

Another NC model can select the maximum P/L to be derived from among the set of multiple minima.

9.2.4. Maximization of Sharpe Ratio

Another NC model can select the maximum Sharpe ratio (after selecting a moving window for calculation) to be derived from among the set of multiple minima.

The standard Sharpe ratio is not the best choice for a cost function. Rather, a “modified” Sharpe ratio (MSR) is used in TRD:

$$\text{MSR} = \begin{cases} \text{profit/drawdown} \\ \text{loss} * \text{drawdown} \end{cases} \quad (46)$$

where drawdown may be replaced some some other measure such as extreme volatility, etc. The value of MSR is that it correctly penalizes losses by multiplying loss by larger drawdown (or volatility) instead of diminishing loss by that amount, which is consistent with the treatment of profit in this cost function.

10. Multiple Trading Systems

TRD is designed to process multiple trading systems. A top-level text parameter file read in by the running code adaptively decides which trading systems to include at any upcoming epoch, without

requiring recompilation of code.

For example, a master controller of system libraries could change this parameter file at any time so that at the next epoch of realtime trading a new set of systems could be in force, or depending on the markets contexts a set of top-level master-controller parameters could decide in training (and used for realtime this way as well) which libraries to use. The flag to include a system is a number which serves as the weight to be used in averaging over signals generated by the systems prior to taking a true position. This approach permits the possibility of encasing all trading systems in a global risk-management and a global optimization of all relevant trading-rule parameters.

TRD is designed to easily insert and run multiple trading systems, e.g., to add further diversification to risk-managing a portfolio. Some trading systems may share indicators and parameters, etc. An example of a CMI trading system is given below.

10.1. Canonical Momenta Indicator (CMI) Trading System

This trading system used to determine the multivariate {Long, Short, Out of Market} market signals at each epoch is based on Canonical Momenta Indicators (CMI). The strengths of CMI as trading indicators are discussed in other publications [11,14,21,22].

For an example of a rationale for developing CMI for trading, consider developing price or return trading rules for break-outs of volatility bands. Examination of Eqs. (3) and (21) shows that break-outs of volatility bands correspond to relatively less probable events than events within these bands. Similarly, the correlation “weight”, $g_{ij}(dy_{dx}^j)$, of $(dy_{dx}^i)^{\dagger}$ in Eq. (21) also can be a measure of such relatively less probable events. Similar arguments can be made for using CMI for trend-following trading rules. For multivariable N -dimensional correlated systems, the CMI provide N such indicators, whereas prices or returns could require a set comparable to $N(N-1)/2$ indicators due to correlations.

Since trading requires taking positions on individual markets (or indices), the best uses of CMI are for price information (close, bid, ask, etc.) in (sub-)sets of markets that, within reasonable time periods, are expected to “typically” have fairly specific degrees of correlation. Then, the CMI can be used as complementary indicators with other indicators.

10.1.1. Consistent Use of Distributions

Previous use of the CMI was determined by adaptively fitting a multivariate Gaussian-Markovian distribution to data, exercising parameterized trading rules on *bona fide* momenta defined by this distribution. Trading rule parameters are defined as windows used for moving averages, scales of volatility used for thresholds to effect long or short positions, scales used for exponential moving averages, stop-losses, etc. The (inverse) covariance matrices used for contract-size risk-management are used as well for the CMI.

The above description of the use of CMI defines Trained trading-rules acting on upcoming patterns of market activity defined by fitted distributions, while contract risk-management is defined by statistical forecasts of these distributions, taking portfolio moments over projected trades.

10.1.1.1. Use of Projected Distributions

As reported in previous CMI publications, sometimes modest enhancements of the use of the distribution for CMI are possible using forecasts of the distribution. In particular, the use of the most-probable estimate of the multivariate distribution [2] can be useful to define windows for moving averages of CMI that include the next forecasted point. In the previous notation,

$$\frac{dy^i}{dt} = g^i - g^{1/2}(g^{-1/2}g^{ij})_j \quad (47)$$

Relatively fast adaptive updates of data on short times scales can use this most-probable equation, instead of the full distribution. Note that utilizing the multivariate metric to update/forecast market data is inconsistent with updating marginal returns in current TRD algorithms.

As reported in previous papers, for adaptive trading rules, the relative volatility of the CMI do not change that much, and so the effect predictor of the change in returns is just the mean. In practice, the fit or

choice of the scales used for moving averages of the CMI define the weights given to these means. If most current information is to be weighted higher than previous information, this can be achieved simply using exponential moving averages instead of simple moving averages. The weighting used for exponential averaging is usually used as a trading-rule parameter in TRD systems.

10.2. Trend & Volatility, Price and CMI, Value-Weighting, Level II, etc.

Some trading systems easily coded into TRD include trend and volatility based indicators and rules, including price and CMI based indicators and rules, and value (volume, time, volatility, etc.) weighted indicators and rules based on these.

A Level II system based on size-weighted and distance-weighted (from traded price) Bid and Ask pressure and momentum on the next traded price is being considered for tick-level trading.

Compiled C code is considered among the fastest codes (e.g., see <http://dada.perl.it/shootout/>), and TRD does several checks when called by any market, before deciding to enter into extended memory allocation, calculations of trading indicators and trading rules, or portfolio optimization, etc. Such checks include whether all markets have picked up orders pending, whether the portfolio has changed since the last bar, etc.

Logic for algorithmic order execution is easily implemented in TRD. For example, Limit orders can be placed as Fill And Kill (or some order logic appropriate to a given platform and exchange), and subsequent broker-filled orders determine whether positions should be modified on the next bar, etc.

10.3. Sets of Multiple Optimal Trading Parameters

Similar to using ASA above, with the MULTI_MIN=TRUE OPTIONS set, to find multiple optimal sets of contracts using the portfolio risk flags in the TRD code, ASA also is set to return multiple optimal sets of trading parameters during Training of trading rules. During Testing and RealTime use of the TRD code to fit trading rules, Long/Short signals are determined by averaging over all optimal sets determined by a threshold of their associated cost functions.

Additional hooks in the code permit trading systems and rules to be set separately for each market.

10.3.1. Recursive ASA Fits

During recursive Training fits, the outermost ASA optimization shell is in the trading-rule space over the portfolio-risk risk space. If portfolio-risk calculations are performed separately at finer time scales than trading-rule optimizations, then the TRD code uses just one best set of trading-rule parameters.

10.4. Adaptive Logic

Weights of systems can be floating point values, not just boolean, to be fit at any time during trading, albeit to be robust reasonable amounts of data should be used between fits. This permits quite different systems, as well as multiple modifications of systems, to be included in fits. These weights can be multiplied by variables, e.g., volatilities of returns, off-diagonal elements of covariance matrices, etc., to provide additional adaptive features.

11. TRD Highlights

The TRD code comprises about 50 files containing about 125 TRD-specific functions in about 15K modified-ASA lines of C code and about 20K TRD-specific lines of C code. (The ASA C code comprises 7 files containing about 50 functions.) The code compiles and runs under gcc or g++ across platforms, e.g., under ThinkPad/XPPPro/Cygwin, Tadpole/SPARC/Solaris, x86/FreeBSD, Linux, etc.

If non-analytic distributions must be transformed to copulas, and also their inverse transformations calculated, this of course requires additional processing power. ASA has hooks for parallel processing, which have been used by people in various institutions. for some years now, and then this additional processing could be better accommodated.

Standard Unix scripts are used to facilitate file and data manipulations. For example, output plots — e.g., 20 sub-plots per page for multiple indicators, positions, prices, etc., versus time of day — for each market

for each day's trading are developed using RDB (a Perl database tool), Gnuplot, and other Unix scripts.

TRD is written to run in Batch for Training and Testing on historical data, and to interface as a function or a DLL call in RealTime with a trading platform, e.g., TradeStation (TS), Fidelity's Active-Trader Wealth-Lab (WL), etc. Batch mode does not require any connection with a RealTime trading platform. In RealTime mode, after checking data, position and order information, the Batch mode code is used to process new orders. I.e., the same core code is used for RealTime, Training and Testing, ensuring that all results are as consistent as possible across these three modes. If no risk-management and no correlations are to be processed, then TRD can reply asynchronously to multiple markets. Applications can be developed, including equities and their indices, futures, Forex, and options.

Full sets of Greeks for multivariate options can be processed using my PATHINT codes, and a faster algorithm PATHTREE has been developed. PATHTREE is a binomial tree to evolve probability distributions defined by general nonlinear Gaussian Markovian processes — multiplicative noise, a published algorithm created by the author and thoroughly tested by a team led by the author [20]. Both PATHTREE and PATHINT have been applied to options codes, e.g., delivering full sets of Greeks based on such underlying probability distributions. Both codes are written in vanilla C. Because of its speed of processing, PATHTREE has been used to fit the shape of distributions to strike data, i.e., a robust bottom-up approach to modeling dependence of strikes on volatilities.

All trading logic and control is in the “vanilla” C TRD code, e.g., not using TS EasyLanguage (EL) or WL WealthScript (WS) code, so it can be used with just about any trading platform, e.g., which compiles and runs without warnings under gcc or g++. For example, TRD interacts with TS using a DLL prepared using gcc or g++ under Cygwin — since other users too have found that a DLL is required to read in information to TS, and with WL using an executable — since other users too have found out that C++ COM wrappers on DLLs do not work well or at all with WL.

11.1. Relative Time Scales of Fits

The above considerations suggest how reasonable relative time scales are defined for Training/Testing vs RealTime use of Trading Rules, Risk Management, and trading systems based on fitted distributions.

On short time scales, a moving window of market data is adaptively fit to a multivariate exponential distribution, e.g., using shortest windows to average over data to calculate covariance/correlation matrices in the Gaussian transformed space as discussed above. These distributions, perhaps also accounting for the influence of the projected most-probable points in the moving windows, are used to calculate indicators upon which trading rules are defined.

Risk-management is performed at longer time scales. E.g., if the short-time scales of intraday trading for adaptively fitting marginal distributions on the order of minutes, then portfolio risk-management might be on the order of days. If the time resolution permits fast computations of adaptive risk management, then contract sizes could be adaptively fit within shorter time windows.

Longer time scales are used to fit trading-rule parameters, e.g., on the order of a week or weeks for intraday trading. Such fits include the adaptive processes of recursive levels of risk management and trading indicators using fitted distributions of moving windows of market data.

For example, the data flow of market data (.dat) and current broker-filled positions (.pos) from TS or WL to TRD, and orders to be filled from TRD to TS or WL (.ord), is handled as:

```
TS/WL (hhmm-1.ord_in, hhmm.dat_out,, hhmm.pos_out)
    n.b.: hhmm may be hhmm-1 depending on execution bar
TRD (hhmm.dat_in, hhmm.pos_in, hhmm.ord_out) [all have same time stamps]
TS/WL (hhmm.ord_in, hhmm+1.dat_out, hhmm+1.pos_out)
```

An adaptive user-defined variable determines the frequency of risk-management calculations to be performed during trading sessions, e.g., between specific trading epochs. At these epochs new contract sizes of markets are calculated for future orders.

11.2. Inter-Day Trading

TRD has an option, DAILY, which trades on daily time scales. This is useful for Training and Testing trading systems designed for trading on daily time scales.

11.3. Post-Processing of Multiple Optima/“Mutations”

It should be understood that any sampling algorithm processing a huge number of states can find many multiple optima. ASA's MULTI_MIN OPTIONS are used to save multiple optima during sampling. Some algorithms might label these states as “mutations” of optimal states. It is important to be able to include them in final decisions, e.g., to apply additional metrics of performance specific to applications. Experience shows that all criteria are not best considered by lumping them all into one cost function, but rather good judgment should be applied to multiple stages of pre-processing and post-processing when performing such sampling.

Within a chosen resolution of future contracts and trading parameters, the huge numbers of possible states to be importance-sampled presents multiple optima and sometimes multiple optimal states. While these can be filtered during sampling with various criteria, it is more useful not to include all filters during sampling, but rather to use ASA's MULTI_MIN OPTIONS to save any desired number of these optimal states for further post-processing to examine possible benefits versus risk according to various desired important considerations, e.g., weighting by correlations, adding additional metrics of performance, etc.

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13. About the Author

Lester Ingber, Ph.D.

Full CV

http://www.ingber.com/ingber_CV.pdf (or [ingber_CV.txt](http://www.ingber.com/ingber_CV.txt))

LinkedIn Profile

<http://www.linkedin.com/in/ingber>

Professional Experience

- Over 100 publications
- Lester Ingber Research (LIR), Interdisciplinary Research/Consulting 1989-
- DUNN Capital Management, Stuart FL, Director R&D 2002-2003
- DRW Trading, Chicago IL, Director R&D 1997-2001
- George Washington University, Research Professor of Mathematics 1989-1990
- National Research Council, Senior Research Associate 1989
- US Army Concepts Analysis Agency, Guest Professor 1989
- Naval Postgraduate School, Professor of Physics 1986-1989
- National Research Council, Senior Research Associate 1985-1986
- Physical Studies Institute, President Nonprofit Corp. 1970-1986
- UC San Diego, Asst. Research Physicist 1970-1972
- State University New York at Stony Brook, Asst. Professor of Physics 1969-1970

Education

- National Science Foundation Postdoc, UC Berkeley and UC Los Angeles 1967-1969
- University of California San Diego, Ph.D. 1966, Theoretical Nuclear Physics
- California Institute of Technology, B.S. 1962, Physics
- Brooklyn Technical High School, Diploma 1958

Published Expertise

- Statistical Mechanics of Financial Markets — Options, Bond Futures, Trading Systems, Risk
- Statistical Mechanics of Neocortical Interactions — Short-Term Memory and EEG
- Statistical Mechanics of Combat — Baselined Simulations to Exercise Data
- Stochastic Algorithms — Simulated Annealing Optimization and Path Integration
- Theoretical Nuclear Physics — Nucleon-Nucleon Scattering, Nuclear Matter, Neutron Stars
- Teaching Methodologies — Private School Developed High-School and College Curricula
- Physics of Karate — Teaching Methodology Leading to 8th-Dan Black Belt

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