

**Statistical Mechanics of Combat (SMC):
Mathematical Comparison of Computer Models to Exercise Data**

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NTC BATTLEFIELD DATA COLLECTION
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PRELIMINARY NTC STUDY
SIMULATION EXTRAPOLATIONS
MATHEMATICAL EXTRAPOLATIONS
CONNECTIONS TO ATTRITION-DRIVEN MODELS
CANONICAL MOMENTA INDICATORS (CMI)
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GENERAL DESCRIPTION

Dated Information

Most of the information in this lecture is dated circa 1990.

The methodology presented is not so dated. Applications have been made, and are being made, to selected projects across several disciplines.

Short Description of Project

The powerful techniques of modern nonlinear statistical mechanics are used to compare battalion-scale combat computer models (including simulations and wargames) to exercise data. This is necessary if large-scale combat computer models are to be extrapolated with confidence to develop battle-management, C³ and procurement decision-aids, and to improve training. This modeling approach to battalion-level missions is amenable to reasonable algebraic and/or heuristic approximations to drive higher-echelon computer models.

Each data set is fit to several candidate short-time probability distributions, using methods of “very fast simulated re-annealing” with a Lagrangian (time-dependent algebraic cost-function) derived from nonlinear stochastic rate equations. These candidate mathematical models are further tested by using path-integral numerical techniques we have developed to calculate long-time probability distributions spanning the combat scenario.

We have demonstrated proofs of principle, that battalion-level combat exercises can be well represented by the computer simulation JANUS(T), and that modern methods of nonlinear nonequilibrium statistical mechanics can well model these systems. Since only relatively simple drifts and diffusions were required, in larger systems, e.g., at brigade and division levels, it might be possible to “absorb” other important variables (C³, human factors, logistics, etc.) into more nonlinear mathematical forms. Otherwise, this battalion-level model should be supplemented with a “tree” of branches corresponding to estimated values of these variables.

We also have applied this methodology to develop and analyze Integrated Strike Warfare on a joint-JANUS(T) model of Navy cruise missile support of Army AirLand battles.

TWO COMPLEMENTARY PROJECT TASKS

Compare JANUS(T) and NTC — Analysis

Model battalion-level combat with Lanchester-type systems

Models drive corps- and theater-level computer-wargames

Seek patterns of combat scenarios

Science = Empirical Data + Models = Statistics + Physics

JANUS-modeling requires explicit identification of NTC data

Sensor/Weapons parameters

MILES PK's

Force structure and movements

Prepare JANUS(T) for NTC — Training

‘What if’ capability

Pre- and post-exercise training for commanders at NTC

Preparation of scenarios by NTC Operations

Experiment with different tactics

Maximize mission accomplishment

Requires JANUS(T)-qualification to fill gaps in NTC data

WHAT IS JANUS(T)?

Input

Physics-Based Simulation

- Specific weapons systems and platforms

- Environment: terrain and weather

- Interactions between systems and environment

Interactive

- Between players and computer program

Stochastic

- Calculates probabilities of detection, acquisition, hit, kill, etc.

Flexible Data Sets

- Research “what-if”

MATHEMATICAL-MODEL COMPARISON PROCESS

JANUS(T) Not Reality

- JANUS(T) algorithm model of human perception, attention, decision-making, etc., is basically serial and logical, not parallel and associative.
- JANUS(T) does not permit direct-fire fratricide.
- A mouse is not a tank to many wargamers.

NTC Not Reality

- NTC constraints, imposed for training, are not necessarily present in combat.
- NTC data includes resurrections, especially of commanders.
- Laser ranges of MILES system (Multiple Integrated Laser Engagement System) are not equivalent to actual weapons' ranges.
- Time-dependent kill data on computer tape can be off at least a factor of two from more accurate "take-home package" information.

Context-Dependence Of Analysis And Methodology

Division-level context + battalion-level attrition equations likely cannot be replaced by division-level attrition equations.

- C^3
- Tactics
- Terrain
- Synchronization at Brigade Level
- Management at Division Level
- Operational Objectives

NTC BATTLEFIELD DATA COLLECTION

Some Problems Qualifying NTC Data

Man-packs with B units enable infantry to be tracked, but one man-pack can represent an aggregate of from a squad to a company.

It is estimated that only about 30% of all possible kills result in possible pairings.

Multiple kills are often noticed on the CIS tape. These are most likely caused by MILES devices malfunctioning and/or by radar-induced errors in transponders.

The PK tables used by MILES devices were obtained by Loral from TRADOC. TRADOC got this data from AMC. In order to declassify the data, highly aggregated averaged PK's were developed.

Qualification of NTC data requires all:

- (1) AAR
- (2) Written part of take-home package (includes OC's documentation)
- (3) ARI/BDM-developed INGRES database
- (4) Full CIS data fed into DEANZA to check holes in data
- (5) Ongoing dialogue with NTC personnel on specific missions

Some NTC Features Relevant to JANUS(T)

Stingers and Dragons are not instrumented. Unfortunately, it appears that especially the Dragons are an important part of the combat.

Vipers require multiple hits (minimum of 3) before the cumulative PK's can register a kill. This is to encourage shooting in volleys.

Good tactics often prevents good data collection. I.e., units may jump in and out of shadows of transmission paths to A stations.

There is no AAR for OPFOR forces, which therefore can't help us to decipher some aspects of their role in combat.

The average movement rates for OPFOR for day/night is about 40/30 km/hr, whereas Soviet doctrine is about 20/12 km/hr.

The opinion that MILES ranges are cut short because of dust and smoke are simply incorrect. The 904 nm wavelength used by the laser has better penetration than visible radiation (but not as good as thermal sensors), so that if a target can be seen, it can very likely be hit. The problem typically arises with TOW's, because troops do not take the required time to boresight their TOW's after each major movement, similar to what is expected in actual combat.

Practically all NTC personnel interviewed stated that good communication seems very often to be more important than good tactics. Eventually, good C³ must be married with JANUS(T) to properly compare NTC and JANUS(T).

MATHEMATICAL PHYSICS APPROACH

Lanchester Theory

Quasi-linear deterministic mathematical modeling is used by many wargames as the primary algorithm to drive the interactions between opposing forces. In its simplest form, this kind of mathematical modeling is known as Lanchester theory:

$$\dot{r} = dr/dt = x_r b + y_r r b$$

$$\dot{b} = db/dt = x_b r + y_b b r$$

where r and b represent Red and Blue variables, and the x 's and y 's are parameters which somehow should be fit to actual data.

Basic Assumption

Attrition per unit time during force on force engagements are a faithful measurable consequence of other aspects of combat, e.g., intelligence, preparation of the battlefield, C³, etc.

This is not true for modern “maneuver warfare” doctrines. This previous attrition study covers only a subset of all aspects of maneuver warfare.

Generalized Lanchester-Type Theory

$$\dot{r} = f_r(r, b) + \sum_i \hat{g}_r^i(r, b)\eta_i$$

$$\dot{b} = f_b(b, r) + \sum_i \hat{g}_b^i(b, r)\eta_i$$

where the \hat{g} 's and f 's are general nonlinear algebraic functions of the variables r and b . The statistical mechanics can be developed for any number of variables, not just two. The η 's are sources of Gaussian-Markovian noise.

In the late 1970's, physicists developed techniques to calculate such nonlinear systems. The first applications was made to neuroscience, then to nuclear physics, now to combat systems.

The inclusion of the \hat{g} 's, called "multiplicative" noise, recently has been shown to very well mathematically and physically model other forms of noise, e.g., shot noise, colored noise, and dichotomic noise.

Math-Physics of One Variable

The Langevin Rate-Equation exhibits a generalized Lanchester equation, wherein drifts can be arbitrarily nonlinear functions, and multiplicative noise is added.

$$M(t + \Delta t) - M(t) \sim \Delta t f[M(t)]$$

$$\dot{M} = \frac{dM}{dt} \sim f$$

$$\dot{M} = f + \hat{g}\eta$$

$$\langle \eta(t) \rangle_{\eta} = 0, \quad \langle \eta(t)\eta(t') \rangle_{\eta} = \delta(t - t')$$

The Diffusion Equation is another equivalent representation of Langevin equations. The first moment “drift” is identified as f , and the second moment “diffusion,” the variance, is identified as \hat{g}^2 .

$$\frac{\partial P}{\partial t} = \frac{\partial(-fP)}{\partial M} + \frac{1}{2} \frac{\partial^2(\hat{g}^2 P)}{\partial M^2}$$

The Path-Integral Lagrangian represents yet another equivalent representation of Langevin equations.

$$P[M_{t+\Delta t}|M_t] = (2\pi \hat{g}^2 \Delta t)^{-1/2} \exp(-\Delta t L)$$

$$L = (\dot{M} - f)^2 / (2 \hat{g}^2)$$

$$P[M_t|M_{t_0}] = \int \cdots \int dM_{t-\Delta t} dM_{t-2\Delta t} \cdots dM_{t_0+\Delta t} \\ \times P[M_t|M_{t-\Delta t}] P[M_{t-\Delta t}|M_{t-2\Delta t}] \cdots P[M_{t_0+\Delta t}|M_{t_0}]$$

$$P[M_t|M_{t_0}] = \int \cdots \int \underline{D}M \exp(-\sum_{s=0}^u \Delta t L_s)$$

$$\underline{D}M = (2\pi \hat{g}_0^2 \Delta t)^{-1/2} \prod_{s=1}^u (2\pi \hat{g}_s^2 \Delta t)^{-1/2} dM_s$$

$$\int dM_s \rightarrow \sum_{\alpha=1}^N \Delta M_{\alpha s}, \quad M_0 = M_{t_0}, \quad M_{u+1} = M_t$$

Path Integral: Long-Time Distribution

Given a form for L , we use the path-integral to calculate the long-time distribution of variables r and b at time t , given values of r and b at time t_0 . This is impossible in general to calculate in closed form, and we therefore must use numerical methods.

The path-integral calculation of the long-time distribution provides an internal check that the system can be well represented as a nonlinear Gaussian-Markovian system. This calculation also serves to more sensitively distinguish among alternative Lagrangians which may approximately equally fit the sparse data.

The use of the path integral to compare different models is akin to comparing short- and long-time correlations.

Complex boundary conditions can be cleanly incorporated into this representation, using a variant of “boundary element” techniques. This typically is not true for the differential-equation representations.

Fitting the Information in the Lagrangian

The Lagrangian must be fitted to empirical data in two nested procedures: Within sets of trial functions, each set must have its parameters/coefficients fitted. Then the probability distribution, considered as a functional of its variables, can be used to describe the evolution of the system.

Similar to work in other disciplines in fitting algebraic forms to search out mechanisms of interactions, much experience needs to be gained, fitting different sets of data with different algebraic forms, to search out functional mechanisms common to specific systems (classes of combat).

Example: Lanchester-Like 2-Variable Model

Consider just simple additive noise:

$$\dot{r} = x_b^r b + y_{br}^r br + z^r \eta_r$$

$$\dot{b} = x_r^b r + y_{rb}^b rb + z^b \eta_b$$

where the z 's are constants. If we assume that the noise η_r is uncorrelated with the noise η_b , then

$$L = \frac{(\dot{r} - x_b^r b - y_{br}^r br)^2}{2z^{r2}} + \frac{(\dot{b} - x_r^b r - y_{rb}^b rb)^2}{2z^{b2}}$$

$$\dot{r} = \Delta r / \Delta t, \Delta r = r(t + \Delta t) - r(t)$$

$$\dot{b} = \Delta b / \Delta t, \Delta b = b(t + \Delta t) - b(t)$$

In terms of L , the probability P of obtaining a change in r and b during Δt is

$$P = (2\pi\Delta t)^{-1} (z^r z^b)^{-1/2} \exp(-L\Delta t)$$

Nonlinear multiplicative noise induces a Riemannian geometry in the space of variables, and more care needs to be taken in defining the Lagrangian. The ideal size of Δt can be derived from the Lagrangian.

Fitting Killer-Victim Scoreboards

For example, consider two Red systems, RT (Red tanks) and $RBMP$, and three Blue systems, BT , $BAPC$ and $BTOW$, where RT specifies the number of Red tanks at a given time t , etc. Consider the kills suffered by BT , ΔBT , e.g., within a time epoch $\Delta t \approx 5$ minutes:

$$\begin{aligned} \frac{\Delta BT}{\Delta t} \equiv \dot{BT} = & x_{RT}^{BT} RT + y_{RT}^{BT} RT BT + x_{RBMP}^{BT} RBMP + y_{RBMP}^{BT} RBMP BT \\ & + z_{BT}^{BT} BT \eta_{BT}^{BT} + z_{RT}^{BT} \eta_{RT}^{BT} + z_{RBMP}^{BT} \eta_{RBMP}^{BT} \end{aligned}$$

where the η 's represent sources of (white) noise (in the Ito prepoint discretization). Equations similar to the \dot{BT} equation are also written for \dot{RT} , \dot{RBMP} , \dot{BAPC} , and \dot{BTOW} .

Very Fast Simulated Re-Annealing (VFSR → ASA)

Very Fast Simulated Re-Annealing (VFSR), now called Adaptive Simulated Annealing (ASA), is an algorithm that fits empirical data to a theoretical cost function over a D -dimensional parameter space, adapting for varying sensitivities of parameters during the fit.

Empirically Fitting Combat Power Scores

For example, the previous five coupled stochastic differential equations can be represented equivalently by a short-time conditional probability distribution, P , in terms of a Lagrangian, L :

$$P(R\cdot, B\cdot; t + \Delta t | R\cdot, B\cdot; t) = \frac{1}{(2\pi\Delta t)^{5/2} \sigma^{1/2}} \exp(-L\Delta t)$$

where σ is the determinant of the inverse of the covariance matrix, the metric matrix of this space, $R\cdot$ represents $\{RT, RBMP\}$, and $B\cdot$ represents $\{BT, BAPC, BTOW\}$.

After the $\{x, y, z, j\}$ are fit to the data, and a mathematical model is selected, another fit can be superimposed to find the effective “combat scores,” defined here as the relative contribution of each system to the specific class of scenarios in question.

Thus, we calculate the aggregated conditional probability

$$\begin{aligned} P_A(r, b; t + \Delta t | R\cdot, B\cdot; t) = & \int dRT dRBMP dBT dBAPC dBTOW \\ & \times P(R\cdot, B\cdot; t + \Delta t | R\cdot, B\cdot; t) \\ & \times \delta(r - w_{RT}^r RT - w_{RBMP}^r RBMP) \\ & \times \delta(b - w_{BT}^b BT - w_{BAPC}^b BAPC - w_{BTOW}^b BTOW) \end{aligned}$$

where the w 's represent the desired combat scores.

Lagrangian Assessment of Dynamics of Combat

Valid at each point $M^G(x, t)$:

$$\text{“Momentum”} = \Pi^G = \frac{\partial L}{\partial(\partial M^G/\partial t)}$$

$$\text{“Mass”} = g_{GG'} = \frac{\partial L}{\partial(\partial M^G/\partial t)\partial(\partial M^{G'}/\partial t)}$$

$$\text{“Force”} = \frac{\partial L}{\partial M^G}$$

$$\text{“} F = ma\text{”}: \delta L = 0 = \frac{\partial L}{\partial M^G} - \frac{\partial}{\partial t} \frac{\partial L}{\partial(\partial M^G/\partial t)}$$

Robustness

Well models other stochastic systems

Unit systems \rightarrow ? combat mesoscales

Possibility (random set) theory may offer some microscopic derivation

Large-scale statistics of large networks

Derivation of Most Probable Path (Trace)

$$\partial P/\partial M^G = 0$$

$$\partial \bar{M}^G/\partial t = g^G - \frac{1}{2} g^{-1/2} \partial(g^{1/2} g^{GG'})/\partial \bar{M}^G$$

not simply deterministic limit of Langevin equations

uncertainty, e.g., intelligence, impacts most probable paths

PRELIMINARY NTC STUDY

Preliminary Mathematical Modeling of NTC Data

U.S. Army Tank/Mechanized Infantry Battalion Task Force (M60, M113, ITV)
Defending Against OPFOR Regimental Attack
“Siberia” Area of National Training Center

From the single NTC trajectory qualified to date, 7 five-minute intervals in the middle of the battle were selected. From six JANUS(T) runs, similar force-on-force time epochs were identified, for a total of 42 data points. In the following fits, r represents Red tanks, and b represents Blue tanks.

Fitting NTC data to an additive noise model, a cost function of 2.08 gave:

$$\dot{r} = -2.49 \times 10^{-5}b - 4.97 \times 10^{-4}br + 0.320\eta_r$$

$$\dot{b} = -2.28 \times 10^{-3}r - 3.23 \times 10^{-4}rb + 0.303\eta_b$$

Fitting NTC data to a multiplicative noise model, a cost function of 2.16 gave:

$$\dot{r} = -5.69 \times 10^{-5}b - 4.70 \times 10^{-4}br + 1.06 \times 10^{-2}(1+r)\eta_r$$

$$\dot{b} = -5.70 \times 10^{-4}r - 4.17 \times 10^{-4}rb + 1.73 \times 10^{-2}(1+b)\eta_b$$

Fitting JANUS(T) data to an additive noise model, a cost function of 3.53 gave:

$$\dot{r} = -2.15 \times 10^{-5}b - 5.13 \times 10^{-4}br + 0.530\eta_r$$

$$\dot{b} = -5.65 \times 10^{-3}r - 3.98 \times 10^{-4}rb + 0.784\eta_b$$

Fitting JANUS(T) data to a multiplicative noise model, a cost function of 3.42 gave:

$$\dot{r} = -2.81 \times 10^{-4}b - 5.04 \times 10^{-4}br + 1.58 \times 10^{-2}(1+r)\eta_r$$

$$\dot{b} = -3.90 \times 10^{-3}r - 5.04 \times 10^{-4}rb + 3.58 \times 10^{-2}(1+b)\eta_b$$

NTC-Model Interpretation

Other systems should be included to represent r and b , especially Red *BMP*'s.

However, the *BTOW* (Blue Tow's) had a surprisingly small contribution relative to the *BT* (Blue tanks). In this particular battle, the Blue forces failed to properly identify and defend against all possible enemy avenues of approach into and through the Blue sector. Red force reconnaissance located the bulk of the Blue force defensive positions prior to the attack, and the Red force scheme of maneuver avoided the Blue positions. Six of seven Blue TOWs were killed, five by Red tanks and one by a Red BMP. Blue TOWs were credited with killing two Red tanks and two other (non-BMP) vehicles.

It is generally observed at NTC that troops do not use their TOW's as much as doctrine and apparent logic dictate. AMSAA tends to agree that a "mistrust" of this system probably stems from the exceptional high degree of accuracy of boresighting required.

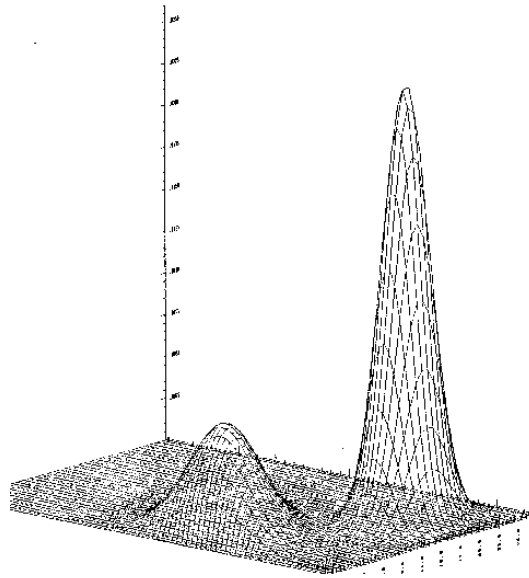
Model Comparison

This comparison illustrates that two different models about equally fit the short-time distribution. The multiplicative noise model shows that about a factor of 100 of the noise might be “divided out,” or understood in terms of the physical log-normal mechanism.

In order to discern which model best fits the data, we turn to the path-integral calculation of the long-time distribution, to see which model best follows the actual data.

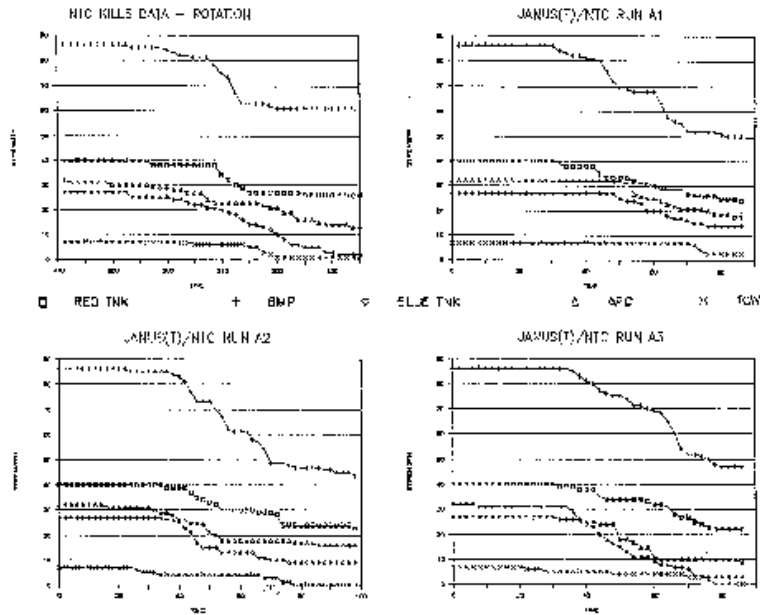
3-D View of Path Integral

In this sample plot, the horizontal axes represent Red and Blue forces. The vertical axis represents the long-time probability of finding values of these forces. In general, the probability will be a highly nonlinear algebraic function, and there will be multiple peaks and valleys. For the JANUS(T)/NTC additive noise case, two time slices are shown superimposed. Taking the initial onset of the force-on-force part of the engagement as 35 minutes on the JANUS(T) clock, these peaks represent 50 and 100 minutes.



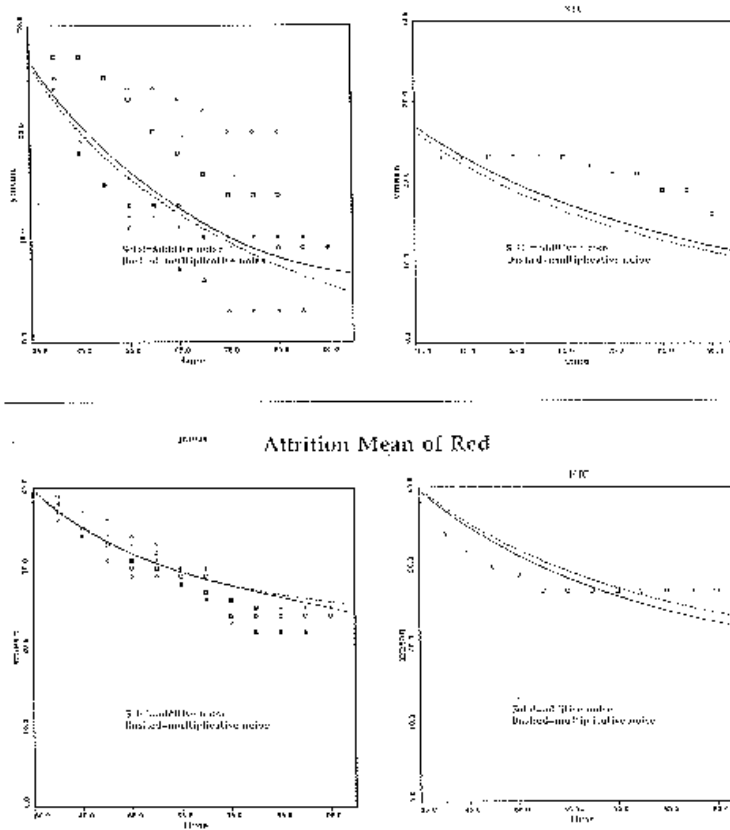
JANUS(T)/NTC Exit Probabilities (Scoreboard)

Exit-probability boundary conditions were used for the zero-force axes, and reflecting boundary conditions were used for the starting-force axes. Results are for the JANUS(T)/NTC additive noise case at 100 minutes.



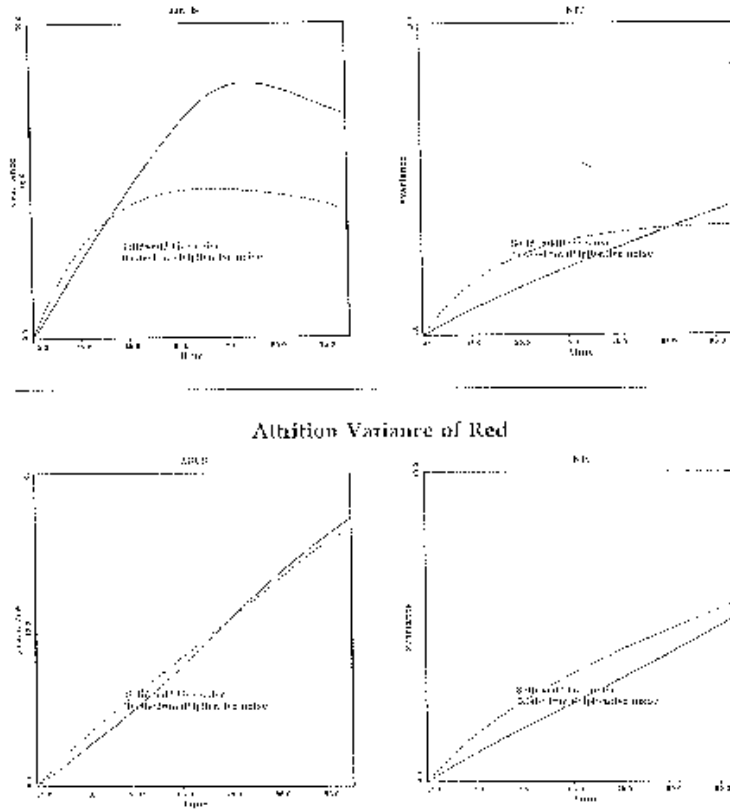
JANUS(T) & NTC Attrition Means

The means of the additive-noise and multiplicative-noise models agree with other very well.



JANUS(T) & NTC Attrition Variances

The Blue JANUS(T) variances serve to distinguish only the additive noise model as being consistent with the JANUS(T) data.



Discussion of Study

Data from 35 to 70 minutes was used for the short-time fit. The path integral used to calculate this fitted distribution from 35 minutes to beyond 70 minutes. This serves to compare long-time correlations in the mathematical model versus the data, and to help judge extrapolation past the data used for the short-time fits.

Note that the means are fit very well by this model, even in out-of-sample time periods, something that other Lanchester modelers have not achieved, especially with such empirical data.

Other Lanchester modelers most often do not consider noise at all, and at best just extract additive noise in the form of regression excesses. More work is required to find a better (or best?) algebraic form. The resulting form is required for input into higher echelon models.

We have demonstrated proofs of principle, that battalion-level combat exercises can be well represented by the computer simulation JANUS(T), and that modern methods of nonlinear nonequilibrium statistical mechanics can well model these systems. Since only relatively simple drifts and diffusions were required, in larger systems, e.g., at brigade and division levels, it might be possible to “absorb” other important variables (C^3 , human factors, logistics, etc.) into more nonlinear mathematical forms. Otherwise, this battalion-level model should be supplemented with a “tree” of branches corresponding to estimated values of these variables.

SIMULATION EXTRAPOLATIONS

Unit Performance from JANUS(T) NTC Surrogate Model

The best resolution presently available from NTC is at the company level.

The JANUS(T) what-if model can provide better resolution, at least statistically consistent with NTC data.

Distinguish between:

- reconnaissance
- active combatants

Distinguish between:

- good shooters
- poor shooters

JOINT TLAM/SLAM PROJECTS

A thesis student studied the use of a battleship battle group deploying Navy Tomahawk C's to support Army-Air Force AirLand Persian Gulf scenarios.

Another thesis student studied the use of carrier battle groups deploying Tomahawk D's, SLAM's and RPV's to support Integrated Strike Warfare (ISW) AirLand inter-German border scenarios.

Working with TRAC-MTRY and TRAC-WSMR personnel, we created these joint scenarios on JANUS(T). The present JANUS(T) construction was the highest-resolution large-scale Navy simulation currently available.

With Navy and Army represented in JANUS(T), we included Marine Corps and Air Force operations. Another previous NPS thesis student included amphibious operations.

We also use statistical mechanics algorithms, developed for our NTC project, for sensitivity analyses of the data provided by these combat computer models.

MATHEMATICAL EXTRAPOLATIONS

Fidelity Issues

Unit Fidelity

math aggregation required for interpretation at multiple scales

Battalion-Brigade Fidelity

this project uses present NTC data

requires ~ company-fidelity data

Division Fidelity

calibrate to brigade-level exercises, soon at NTC?

requires ~ battalion-fidelity data

Human Factors Issues

Poor Representation in Computer Models

“Absorb” Factors in Fitted SDE Coefficients?

seems o.k. at battalion fidelity

absorbed in force on force coefficients

must test by extrapolating across battles

o.k. at division fidelity?

Other Issues

Representation of C³?

- synchronization at brigade level
- management at division at division level

Representation of Combat Service Support?

Representation of IPB (Intelligence, Preparation of Battlefield)

- (1) Sensitivity of theater models to different approaches
- (2) Inclusion/absorption of human factors into variables/parameters
- (3) Modern systems, e.g., cruise missiles
 - high fidelity interactions
 - short reaction times
 - large spatial coverage
 - requires C³I at multiple scales
- (4) Statistical comparison of approaches
- (5) Baselineing of approaches to some reality

Super-Variables for Theater Models

Specify Super-Variables

(I...) Level of combat, e.g., battalion-brigade

(A...) Terrain

(1...) Force structure

(a...) C³

(i...) IPB

CONNECTIONS TO ATTRITION-DRIVEN MODELS

Basic Assumption at Battalion Level

Dynamic Attrition = Faithful Measure of All Combat Variables

After Fitting Battalion-Level Data

Decision Rules + Local Linearization to Lanchester

reasonable compromise to run wargames in real-time

might neglect noise to feed deterministic models

Applications

Higher-Echelon Army Models

 Theater-Level Context + Battalion-Level Attrition Equations

C³ Models Driven by Attrition Equations

 connect JANUS(T) to higher echelon C³

 degrade/jam communications between wargamers

Reduction to Other Math-Physics Modeling

Phase Transitions

bifurcation develops in path-integral calculation
already calculated in other systems

Chaos

advantages of using algebraic models fitted to data
investigate opportunities to induce chaos in opponents
folding of path integral develops attractors in presence of noise
further reduction to deterministic limit
examine deterministic rate equations
uncertainty certainly exists in combat
but does chaos?

Catastrophe Theory

time-slice of Taylor-expanded/approximated Lagrangian
examine polynomials of $\{r, b\}$ variables, as parameters change

AI complementary to Physics

Physics: codifies knowledge in fitted algebraic forms
AI: decision rules effectively manage this knowledge

Spatial-temporal approximates to present approach

additive-noise limit of multiplicative noise
neglect noise: deterministic partial-differential rate equations
expand Lagrangian as polynomial: Lanchester theory

Predator-Prey Biological Models

interesting but not necessarily relevant
war is predator-predator

CANONICAL MOMENTA INDICATORS (CMI)

New Indicators of Combat

$$\text{Momentum} \equiv \Pi^G = \frac{\partial \underline{L}_F}{\partial(\partial M^G/\partial t)} ,$$

We expect the CMI and the fitted coefficients to be more valuable predictors of events in combat, as the battlefield becomes more nonlinear. We have described a reasonable approach to quantitatively measuring this nonlinearity, and a reasonable approach to faithfully presenting this information to commanders in the field so that they may make timely decisions.

Model

Consider a scenario taken from our NTC study: two Red systems, Red T-72 tanks (RT) and Red armored personnel carriers ($RBMP$), and three Blue systems, Blue M1A1 and M60 tanks (BT), Blue armored personnel carriers ($BAPC$), and Blue tube-launched optically-tracked wire-guided missiles ($BTOW$), where RT specifies the number of Red tanks at a given time t , etc. Consider the kills suffered by BT , ΔBT , e.g., within a time epoch $\Delta t \approx 5$ min

$$\frac{\Delta BT}{\Delta t} \equiv \dot{BT} = x_{RT}^{BT} RT + y_{RT}^{BT} RT BT$$

$$+ x_{RBMP}^{BT} RBMP + y_{RBMP}^{BT} RBMP BT$$

Here, the x terms represent attrition owing to point fire; the y terms represent attrition owing to area fire. The version of Janus(T) used to generate this data does not permit direct-fire fratricide; such terms are set to zero. In most NTC scenarios fratricide typically is negligible.

Now consider sources of noise, e.g., that at least arise from PD, PA, PH, PK, etc. Furthermore, such noise likely has its own functional dependencies, e.g., possibly being proportional to the numbers of units involved in the combat. For simplicity here, still generating much nonlinearity, only diagonal noise terms are considered. Coupling among the variables takes place in the drift terms (deterministic limit); for simplicity only linear terms in the drifts are taken for this prototype study.

$$\frac{\Delta BT}{\Delta t} \equiv \dot{BT} = x_{RT}^{BT} RT + y_{RBMP}^{BT} RT BT$$

$$+ x_{RBMP}^{BT} RBMP + y_{RBMP}^{BT} RBMP BT$$

$$+ z_{BT}^{BT} BT \eta_{BT}^{BT}$$

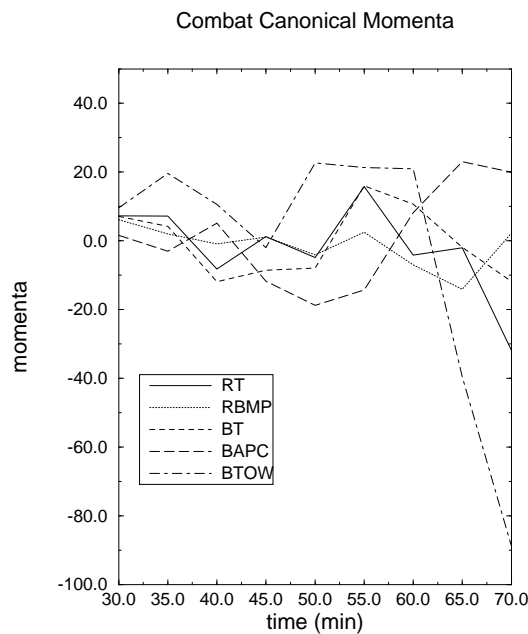
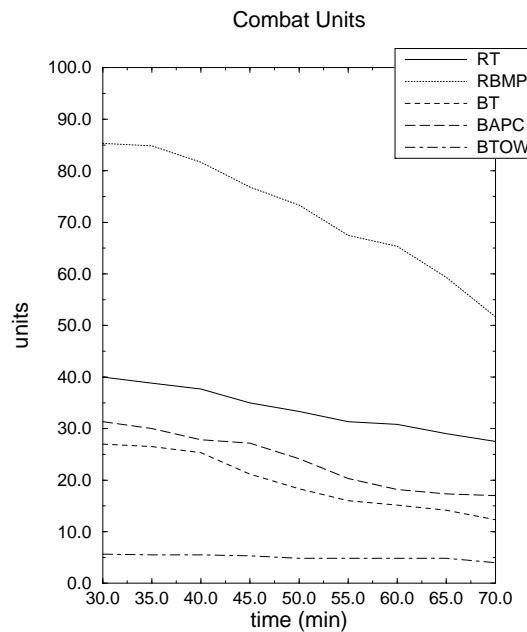
Fitted Model

The table gives the results of ASA fits of the above 5 coupled equations to Janus-generated data. Note that the noise coefficient is roughly the same for all units, being largest for *BTOW*. Note the relative importance of coefficients in “predicting” the immediate next epoch, with *BTOW* larger than *BAPC* larger than *BT* in depleting Red forces (but being multiplied by the total number of units at any time). The coefficients of “prediction” of attrition by Red forces has *RT* larger than *RBMP* against *BTOW*, and *RT* less than *RBMP* against *BT* and *BAPC* (but being multiplied by the total number of units at any time).

	<i>RT</i>	<i>RBMP</i>	<i>BT</i>	<i>BAPC</i>	<i>BTOW</i>	$\eta[.]$
\dot{RT}	-	-	-8.6E-5	-5.9E-3	-3.6E-2	3.7E-3
\dot{RBMP}	-	-	-2.7E-3	-2.2E-2	-3.1E-2	4.3E-3
\dot{BT}	-6.7E-4	-4.7E-3	-	-	-	7.9E-3
\dot{BAPC}	-1.0E-4	-4.0E-3	-	-	-	6.7E-3
\dot{BTOW}	-2.1E-3	-1.2E-6	-	-	-	1.3E-2

JANUS Attrition & CMI

The upper graph in the figure gives the attrition data. The attrition data is given as the average over 6 runs for each time point. The lower figure gives the derived CMI. After the ASA fits, the CMI are calculated for each point in time in each of the 6 runs. The figure gives the average over the 6 runs for each time point. Note that the attrition rate of all units is fairly constant, and so there are no surprises expected in this kind of analysis. The marked changes of the systems at the end of the epoch signals the essential ending of the combat.



TO DO

1997 JANUS Update

Not only are we moving to a new era in tactics and doctrine with the theory of a nonlinear battlefield, and the “next wave” of warfare (information warfare), we’ve also seen a complete turn around in the capabilities of “Red” (old Soviet Union and client states) versus “Blue” (U.S./Nato) forces. When we did the studies of NTC and JANUS data in the late 1980’s the Blue side was at a distinct technological disadvantage and the NTC scenarios were played out that way - the MILES sensors on T72s were positioned so that the T72s could not be killed by frontal hits by any U.S. weapons, while M60s could be killed by any hits from the T72 and just about anything on the battlefield could kill a U.S. APC or TOW vehicle.

In the 1990s the U.S. and NATO have advanced to a new generation of combat systems (M1A2 tank and Bradley Fighting Vehicle) while potential adversaries equipped with “Red” equipment (T72 and BMP) have not.

Attrition Vs Maneuver Warfare

The ‘non-linear’ battle field and the Army’s modern ‘maneuver warfare’ doctrines call for the a switch in emphasis from the fire and maneuver described in Air-Land Battle Doctrine, which carries a connotation of ‘attrition warfare’ to an emphasis on the use of more pure maneuver to when ever possible by-pass and make irrelevant enemy strengths.

The issue is that the strongest proponents of maneuver warfare may consider ‘force ratios’, ‘kill ratios’, ‘attrition rates,’ etc., the tools of poor commanders who should be concentrating on finding and tipping the enemy center of gravity by maneuver rather than calculating ratios for head-on attacks.

While maneuver is the technique of choice, and driving the enemy from the battlefi eld without firing a shot is the goal, in all likelihood even the most masterfully maneuvered force will still fight engagements and battles during a campaign, and knowledge of ratios is still a valid command tool. Given the complex operation environment envisioned on a non-linear battlefi eld, some attrition combat is likely to be taking place at any given time that maneuver is being exercised on another portion of the battlefi eld.

Value of Models and Simulations

We expect the CMI and the fitted coefficients to be more valuable predictors of events in combat, as the battlefield becomes more nonlinear.

Given the high cost of major field exercises in an environment of shrinking budgets, our forces will rely more and more heavily on modeling and simulation to develop, test, and practice tactics and doctrine at all levels. Modeling and simulation remain highly useful devices for making tactical mistakes and learning lessons at little or no cost. The use of CMI and ASA to evaluate and improve these models and simulation remains a worthy goal.