STATISTICAL MECHANICS OF COMBAT WITH HUMAN FACTORS

Lester Ingber
Science Transfer Corporation
P.O. Box 857
McLean, VA 22101

and

David D. Sworder
Department of Applied Mechanics and Engineering Science
University of California at San Diego
La Jolla, CA 92093

Abstract

This highly interdisciplinary project extends previous work in combat modeling and in control-theoretic descriptions of decision-making human factors in complex activities. A previous paper has established the first theory of the statistical mechanics of combat (SMC), developed using modern methods of statistical mechanics, baselined to empirical data gleaned from the National Training Center (NTC). This previous project has also established a JANUS(T)-NTC computer simulation/wargame of NTC, providing a statistical ‘‘what-if’’ capability for NTC scenarios. This mathematical formulation is ripe for control-theoretic extension to include human factors, a methodology previously developed in the context of teleoperated vehicles. Similar NTC scenarios differing at crucial decision points will be used for data to model the influence of decision making on combat. The results may then be used to improve present human factors and C^2 algorithms in computer simulations/wargames.

Our approach is to ‘‘subordinate’’ the SMC nonlinear stochastic equations, fitted to NTC scenarios, to establish the zeroth order description of that combat. In practice, an equivalent mathematical-physics representation is used, more suitable for numerical and formal work, i.e., a Lagrangian representation. Theoretically, these equations are nested within a larger set of nonlinear stochastic operator-equations which include C^3 human factors, e.g., supervisory decisions. In this study, we propose to perturb this operator theory about the SMC zeroth order set of equations. Then, subsets of scenarios fit to zeroth order, originally considered to be similarly degenerate, can be further split perturbatively to distinguish C^3 decision-making influences. New methods of Very Fast Simulated Re-Annealing (VFSR), developed in the previous project, will be used for fitting these models to empirical data.

1. Rationale and Motivation

1.1. INTRODUCTION

In many complex activities involving interactions between human and inanimate systems, e.g., modern combat, the nonlinear synergies capable between these systems make it impossible to separate their influences from the total scenario. However, their relationships and functional dependencies still might be amenable to explicit scientific description.

For example, if $h(t)$ could be determined to be a (time-dependent) human factor, and if $x(t)$ could be determined to be an inanimate factor, then one could imagine that a ‘‘cost function,’’ C, fitting data from a specific class of combat scenarios could be fit by a probability distribution, $P[C]$, (emphasizing the uncertainty and ‘‘noise’’ in both systems). For specificity, consider the completely arbitrary distribution selected only for purposes of illustration:

$$P[C] = N \exp(-C)$$

$$C(x, h; t) = \frac{(dx/dt + 3.04xh^2 + 0.21x^2)^2}{(2.84 + 0.133h^2x^2)} + \frac{(dh/dt + 5.10x^2 + 1.13hx^2)^2}{(4.21 + 0.071h^2x^2)} \qquad (1.1)$$

where $N(x, h; t)$ is a normalization factor for this distribution.

In fact, in a previous paper (IFW) [1], we have derived similar stochastic nonlinear forms, in terms of nonlinear stochastic time-dependent inanimate combat variables (tank attrition, etc.), that fit quite well to data from exercises at the National Training Center (NTC). We propose here to more explicitly include the human-factor variables relevant to decision-making processes at NTC. The determination of such a cost function permits the accurate derivation of graphical aids to visualize the sensitivity of combat macro-variables (force, mass, momentum) as a function of the human decision-making process [2].

It is perhaps just as important for us to clearly state what we are not proposing. We are not proposing that the human-factors variables we will derive, e.g., in the sense of h in Eq. (1.1), will be explicit representations of cognitive activity—such as attentional processes. Rather, these variables are to be considered meta-variables representing the behavioral characteristics of human decision-making [3], in the context of specific NTC scenarios. The above sensitivity measures of the decision-making process perhaps comes closest with this methodology to explicit identification of human factors.

We believe that we can deliver normative (probabilistic) standards of a class(es) of NTC scenarios, by which specific unit decision-making performance can be gauged within this context. Furthermore, our explicit representations of these human factors permits these equations to be used directly in many combat computer models, thereby increasing the utility of these computer models for training and analysis. The lack of human-factors algorithms in combat computer models is notorious, perhaps nefarious [4-6].

Our major thrust will be to identify and to interpret reasonable functional forms much more extensive and detailed than Eq. (1.1). This project is new research territory and it will require extensive and intensive interaction between accumulating critical analyses and accumulating experience with operations at NTC.

The inclusion of human factors in a single equation like Eq. (1.1) is too naive to capture the essence of human decision-making, even if we generalize h and x to include many variables from each opponent, e.g., $\tilde{h}$ and $\tilde{x}$ vectors. To include sharp bifurcations, e.g., alternative branching scenarios due to the perceptions of commanders and actions thereby taken at critical times in combat, we plan to fit a ‘‘tree’’ of distributions like Eq. (1.1), each branch representing an alternative scenario.

Our rationale for this attempt, to generalize our previous NTC fits of inanimate variables [2], is based on our other work modeling the human decision-maker controlling teleoperated robotic vehicles, a decision-making process that is conceptually similar to the role of the commander in combat [3]. We should thereby gain greater fidelity in our fits to NTC data by more explicitly including human factors.

A major thrust of our research will be to expand the linearized theory beyond that currently formulated [3], to include more robust nonlinear features of the underlying theory of human decision making. This approach is now possible because of the spin-off work in the previous project [2], i.e., developing a methodology of Very Fast Simulated Re-Annealing (VFSR) to fit such nonlinear multivariate stochastic models [7].

1.2. SIGNIFICANT ASPECTS TO BE STUDIED

We believe we are addressing the following issues:

(1) Human factors, especially in combat, are nonlinear. Nonlinearity arises for many reasons, ranging from synergies of human factors with physical systems, to multivalued decision trees depicting future states. The inclusion of realistic movements on realistic terrain typically presents a nonlinear spatial-temporal surface on which the variables evolve.

(2) Human factors are stochastic. There are relatively separable influences on decision making, e.g., probabilities associated with detections, acquisitions, hits, and kills. Furthermore, especially in a given complex situation, not only will different people often make different decisions at different times, but, given the same opportunities, the same person often will make different decisions at different times. Therefore, we need multiple runs of similar situations in order to deduce these distributions. Such sets of data, albeit not ideal, is present at NTC.

(3) Human factors are typically observed as ‘‘meta-variables’’ of human performance. Especially because information possessed by decision makers is often incomplete or known to be at least partially incorrect, decision makers must make their decisions based on their current perceptions, their extrapolations to future perceptions, their perceptions of their opponent’s perceptions, etc. These possibilities give rise to alternate behavioral states, in part contributing the nonlinearities and stochasticities discussed above.

(4) Human factors are very context- and domain-dependent. Other approaches to human factors, e.g., in the field of Artificial Intelligence, are also converging on this realization. Analogical reasoning is often more efficient than logical deduction [8].

(5) Even combat models with reasonable combat algorithms do not have reasonable human factors algorithms. Especially because of real-time constraints, these computer models require relatively simple functional relationships if they are to include human factors at all. Our relatively robust stochastic nonlinear approach permits us to identify (multiple) ranges of likely probable states, which then can be approximated by quasi-linear algebraic forms in each range. This forms the basis of an ‘‘expert’’ system which derives knowledge from objective fits of theoretical models to empirical data.

1.3. OUTLINE OF PAPER

Section 2 presents the description of the methodology used in the previous IFW paper. There, background is given of the use of NTC data, development of the JANUS(T)/NTC combat simulation, mathematical techniques, and numerical techniques and results. However, this paper has been written to be relatively self-contained.

Section 3 presents the mathematical background and description of human factors which we believe is relevant to this paper.

The purpose of Section 2 is to demonstrate, not only that aggregated combat has already been successfully modeled with stochastic differential equations in a previous NTC study, but also that the methodology and numerical techniques developed for that study are directly transferable to this expanded human factors project.

The purpose of Section 3 is to present reasonably technical arguments that indeed human-factors aspects of combat, i.e., the behavior of the commander, can be approached by the use of such stochastic differential equations.

It is relevant to this paper to note that the projects reported in Section 2, modeling NTC and JANUS(T), and in Section 3, modeling teleoperated vehicles, both have brought powerful mathematical machinery to bear to the stages of numerical specificity with state-of-the-art successful description of realistic empirical data.

2. Mathematical and Computer Modeling of NTC

2.1. QUALIFICATION OF NTC DATA

2.1.1. Individual Performance

It is clear that the individual performance is extremely important in combat [9], ranging in scale from battle management of the commander, to battle leadership of sub-commanders, to the degree of participation of individual units, to the more subtle degradation of units performing critical tasks.

Our analyses of NTC data concludes that data collected to date is not sufficient to accurately statistically judge individual performance across these scales. However, we do believe that this data is sufficient to analyze battle management, perhaps battle leadership, at the company or platoon level, in some cases, e.g., reflecting the influence of a human commander.

It is important to recognize and emphasize the necessity of improving data collection at NTC, to permit complementary analyses of human factors at finer scales than our statistical approach permits.

Therefore, as understood from experience in simulating physics systems, many trajectories of the ‘‘same’’ stochastic system must be aggregated before a sensible resolution of averages and fluctuations can be ascertained. Given two scenarios that differ in one parameter, and given a sufficient number of trajectories of each scenario, then the sensitivity to changes of a ‘‘reasonable’’ algebraic function to this parameter can offer some analytic input into decisions involving the use of this parameter in combat scenarios. NTC is the best source of such data, albeit it is sparse.

2.1.2. Description of NTC

The U.S. Army National Training Center (NTC) is located at Fort Irwin, just outside of Barstow, California. There have been about 1/4 million soldiers in 80 brigade rotations at NTC, at about the level of 2 battalion task forces (typically about 3500 soldiers and a battalion of 15 attack helicopters), which train against 2 opposing force (OPFOR) battalions resident at NTC. NTC comprises about 2500 km^2, but the current battlefield scenarios range over about 5 km linear spread, with a maximum lethality range of about 3 km. NTC is gearing up for full brigade-level exercises.

Observer-controllers (OC) are present at various levels of command down to about platoon level. A rotation will have three force-on-force missions and one live-fire mission. OPFOR platoon- and company-level personnel are trained as US Army soldiers; higher commanders practice Soviet doctrine and tactics. An OPFOR force typically has ~100 BMP’s and ~40 T72’s.

The primary purpose of data collection during an NTC mission is to patch together an after-action review (AAR) within a few hours after completion of a mission, giving feedback to a commander who typically must lead another mission soon afterwards. Data from the field, multiple integrated laser engagement system (MILES) devices, audio communications, OC’s, and stationary and mobile videocameras, is sent via relay stations back to a central command center where this all can be recorded, correlated and abstracted for the AAR. Within a couple of weeks afterwards, a written review is sent to commanders, as part of their NTC take-home package. It presently costs about 4 million dollars per NTC rotation, 1 million of which goes for this computer support.

There are 460 MILES transponders available for tanks for each battle. The ‘‘B’’ units have transponders, but most do not have transmitters to enable complete pairings of kills-targets to be made. (New MILES devices being implemented have transmitters which code their system identification, thereby greatly increasing the number or recordings of pairings.) Thus, MILES’s without transmitters cannot be tracked. Man-packs with B units enable these men to be tracked, but one man-pack can represent an aggregate of as much as 5 people.

B units send data to ‘‘A’’ stations (presently 48, though 68 can be accommodated), then collected by two ‘‘C’’ stations atop mountains, and sent through cables to central VAX’s forming a core instrumentation system (CIS). There is a present limitation of 400 nodes in computer history for video tracking (but 500 nodes can be kept on tape). Therefore, about 200 Blue and 200 OPFOR units are tracked.

By varying the laser intensity and focusing parameters, a maximum laser-beam spread is achieved at the nominal range specified by the Army. A much narrower beam can reach as far as the maximum range. Focusing and attenuation properties of the laser beam makes these nominal and maximum ranges quite sharp, with resolution supposedly considerably less than several hundred meters under ideal environmental conditions. For example, a weapon might send out a code of 8 words (spaced apart by nsecs), 2 of which must register on a target to trigger the Monte Carlo routine to calculate a PK. Attenuation of the beam past its preset range means that it rapidly becomes unlikely that 2 words will survive to reach the target.

With increasing demands to make training more realistic, the MILES devices need to be upgraded. E.g., degradation of the laser beam under conditions of moderate to heavy smoke and dust might be at least partially offset by sending fewer words per message. New sensor abilities to encode specific shooters will also greatly aid data collection.

It should be understood that present training problems at NTC, e.g., training commanders—especially at Company level—to handle synchronization of more than three tasks, misuse of weapons systems, etc., overshadow any problems inherent in the MILES systems. We repeatedly have expressed this view for well over a year, after going to NTC several times; but only at a recent meeting at Carlisle Barracks, PA, on 17 May 1989, when various school commanders briefed GEN Maxwell Thurman, TRADOC Commander, was this view broadly accepted.

Therefore, to the degree possible in this project, our wargaming efforts should strive to place commanders under these constraints of current interest, e.g., under requirements to synchronize the timing of the movement or repositioning of forces, request for supporting fires (artillery, air strike, etc.), initiation of fires into target zones, the shifting of supporting fires, the execution of local counter-attacks, etc.

2.1.3. Qualification Process

Missing unit movements and initial force structures were completed in the NTC database, often making ‘‘educated guesses’’ by combining information on the CIS tapes and the written portion of the take-home package.

This project effectively could not have proceeded if we had not been able to automate transfers of data between different databases and computer operating systems. One of LI’s students, CPT Mike Bowman, USA, wrote a thesis on the management of the many information-processing tasks associated with this project [10]. He has coordinated and integrated data from NTC, TRADOC (Training and Doctrine Command) Analysis Command (TRAC) at White Sands Missile Range, NM (TRAC-WSMR) and at Monterey, CA (TRAC-MTRY) for use by MAJ Hirome Fujio, USA, for JANUS(T) wargaming at TRAC-MTRY, and for use by Dr. Mike Wehner at Lawrence Livermore National Laboratory (LLNL) Division B, and LI for JANUS(T) and NTC modeling.

2.2. DEVELOPMENT OF JANUS(T)/NTC

2.2.1. Description of JANUS(T)

JANUS(T) is an interactive, two-sided, closed, stochastic, ground combat computer simulation (recently expanded to air and naval combat under several projects projects with LI’s previous NPS thesis students).

Interactive refers to the the fact that military analysts (players and controllers) make key complex decisions during the simulation, and directly react to key actions of the simulated combat forces. Two-sided (hence the name Janus of the Greek two-headed god) means that there are two opposing forces simultaneously being directed by two set of players. Closed means that the disposition of the enemy force is not completely known to the friendly forces. Stochastic means that certain events, e.g., the result of a weapon being fired or the impact of an artillery volley, occur according to laws of chance (random number generators and tables of probabilities of detection (PD), acquisition (PA), hit (PH), kill (PK), etc.). The principle modeling focus is on those military systems that participate in maneuver and artillery operations. In addition to conventional direct fire and artillery operations, JANUS(T) models precision guided munitions, minefield employment and breaching, heat stress casualties, suppression, etc.

Throughout the development of JANUS(T), and its Janus precursor at Lawrence Livermore National Laboratory, extensive efforts have been made to make the model ‘‘user friendly,’’ thereby enabling us to bring in commanders with combat experience, but with little computer experience, to be effective wargamers. There is now a new version, Janus(A), bringing together the strengths of these predecessors.

2.3. DEVELOPMENT OF MATHEMATICAL METHODOLOGY

2.3.1. Background

Aggregation problems in such nonlinear nonequilibrium systems typically are ‘‘solved’’ (accommodated) by having new entities/languages developed at these disparate scales in order to efficiently pass information back and forth [1, 11, 12]. This is quite different from the nature of quasi-equilibrium quasi-linear systems, where thermodynamic or cybernetic approaches are possible. These approaches fail for nonequilibrium nonlinear systems.

In the late 1970’s, mathematical physicists discovered that they could develop statistical mechanical theories from algebraic functional forms

$$\dot{r} = f_r(r, b) + \sum_i \hat{g}_r^i(r, b)\eta_i$$

$$\dot{b} = f_b(b, r) + \sum_i \hat{g}_b^i(b, r)\eta_i \tag{2.1}$$

where the $\hat{g}$’s and f’s are general nonlinear algebraic functions of the variables r and b [13-18]. The f’s are referred to as the (deterministic) drifts, and the square of the $\hat{g}$’s are related to the diffusions (fluctuations). In fact, the statistical mechanics can be developed for any number of variables, not just two. The η’s are sources of Gaussian-Markovian noise, often referred to as ‘‘white noise.’’ The inclusion of the $\hat{g}$’s, called ‘‘multiplicative’’ noise, recently has been shown to very well mathematically and physically model other forms of noise, e.g., shot noise, colored noise, dichotomic noise [19-21].

The ability to include many variables also permits a ‘‘field theory’’ to be developed, e.g., to have sets of (r, b) variables (and their rate equations) at many grid points, thereby permitting the exploration of spatial-temporal patterns in r and b variables. This gives the possibility of mathematically modeling the dynamic interactions across a large terrain.

These new methods of nonlinear statistical mechanics only recently have been applied to complex large-scale physical problems, demonstrating that empirical data can be described by the use of these algebraic functional forms. Success was gained for large-scale systems in neuroscience, in a series of papers on statistical mechanics of neocortical interactions (SMNI) [22-28], and in nuclear physics [29, 30]. These have been proposed for problems in C^3 [1, 2, 11].

Thus, now we can investigate various choices of f’s and $\hat{g}$’s to see if algebraic functional forms close to the Lanchester forms can actually fit the data. In physics, this is the standard phenomenological approach to discovering and encoding knowledge and empirical data, i.e., fitting algebraic functional forms which lend themselves to physical interpretation. This gives more confidence when extrapolating to new scenarios, exactly the issue in building confidence in combat computer models.

The utility of these algebraic functional forms in Eq. (2.1) goes further beyond their being able to fit sets of data. There is an equivalent representation to Eq. (2.1), called a ‘‘path-integral’’ representation for the long-time probability distribution of the variables. This short-time probability distribution is driven by a ‘‘Lagrangian,’’ which can be thought of as a dynamic algebraic ‘‘cost’’ function. The path-integral representation for the long-time distribution possesses a variational principle, which means that simple graphs of the algebraic cost-function give a correct intuitive view of the most likely states of the variables, and of their statistical moments, e.g., heights being first moments (likely states) and widths being second moments (uncertainties). Like a ball bouncing about a terrain of hills and valleys, one can quickly visualize the nature of dynamically unfolding r and b states.

Especially because we are trying to mathematically model sparse and poor data, different drift and diffusion algebraic functions can give approximately the same algebraic cost-function when fitting short-time probability distributions to data. The calculation of long-time distributions permits a clear choice of the best algebraic functions, i.e., those which best follow the data through a predetermined epoch of battle. Thus, dynamic physical mechanisms, beyond simple ‘‘line’’ and ‘‘area’’ firing terms, can be identified. Afterwards, if there are closely competitive algebraic functions, they can be more precisely assessed by calculating higher algebraic correlation functions from the probability distribution.

It must be clearly stated that, like any other theory applied to a complex system, these methods have their limitations, and they are not a panacea for all systems. For example, probability theory itself is not a complete description when applied to categories of subjective ‘‘possibilities’’ of information [31, 32]. Other non-stochastic issues are likely appropriate for determining other types of causal relationships, e.g., the importance of reconnaissance to success of missions [33]. These statistical mechanical methods appear to be appropriate for describing stochastic large-scale combat systems. The details of our studies will help to determine the correctness of this premise.

As discussed previously, the mathematical representation most familiar to other modelers is a system of stochastic rate equations, often referred to as Langevin equations. From the Langevin equations, other models may be derived, such as the times-series model and the Kalman filter method of control theory. However, in the process of this transformation, the Markovian description typically is lost by projection onto a smaller state space [34, 35]. This work only considers multiplicative Gaussian noise, including the limit of weak colored noise [20]. These methods are not conveniently used for other sources of noise, e.g., Poisson processes or Bernoulli processes. It remains to be seen if multiplicative noise can emulate these processes in the empirical ranges of interest, in some reasonable limits [21]. At this time, certainly the proper inclusion of multiplicative noise, using parameters fit to data to model general sources of noise, is preferable to improper inclusion or exclusion of any noise.

2.3.2. Mathematical Methodology

2.3.2.1. Model Development

Consider a scenario taken from our NTC study: two Red systems, RT (Red tanks) and $RBMP$, and three Blue systems, BT, $BAPC$ and $BTOW$, where RT specifies the number of Red tanks at a given time t, etc. Consider the kills suffered by BT, ΔBT, e.g., within a time epoch $\Delta t \approx 5$ minutes:

$$\frac{\Delta BT}{\Delta t} \equiv \dot{BT} = x^{BT}_{RT} RT + y^{BT}_{RT} RT\ BT + x^{BT}_{RBMP} RBMP + y^{BT}_{RBMP} RBMP\ BT$$

$$+ z^{BT}_{BT} BT \eta^{BT}_{BT} + z^{BT}_{RT} \eta^{BT}_{RT} + z^{BT}_{RBMP} \eta^{BT}_{RBMP} \quad (2.2)$$

where the η’s represent sources of (white) noise (in the Ito prepoint discretization). Here, the x terms represent attrition due to point fire; the y terms represent attrition due to area fire; the diagonal z term (z^{BT}_{BT}) represents uncertainty associated with the *target BT*, and the off-diagonal z terms represent uncertainty associated with the *shooters RT* and *RBMP*. The x’s and y’s are constrained such that each term is bounded by the mean of the Killer-Victim Scoreboard (KVS), averaged over all time and trajectories of similar scenarios; similarly, each z term is constrained to be bounded by the variance of the KVS.

Note that the functional forms chosen are consistent with current perceptions of aggregated large-scale combat. I.e., these forms reflect point and area firing; the noise terms are taken to be log-normal (multiplicative) noise for the diagonal terms and additive noise for the off-diagonal terms. The methodology presented here can accommodate any other nonlinear functional forms, and any other variables which can be reasonably represented by such rate equations, e.g., expenditures of ammunition or bytes of communication [11]. Variables which cannot be so represented, e.g., terrain, C^3, weather, etc., must be considered as ‘‘super-variables’’ which specify the overall context for the above set of rate equations.

Equations similar to the $\dot{BT}$ equation are also written for $\dot{RT}$, $\dot{RBMP}$, $\dot{BAPC}$, and $\dot{BTOW}$. Only x’s and y’s which reflect possible non-zero entries in the KVS are free to be used for the fitting procedure. For example, since JANUS(T) does not permit direct-fire fratricide, such terms are set to zero. In most NTC scenarios, fratricide typically is negligible. Non-diagonal noise terms give rise to correlations in the covariance matrix. Thus, we have

$$M^G = \{RT, RBMP, BT, BAPC, BTOW\}$$

$$\dot{M}^G = g^G + \sum_i \hat{g}^G_i \eta^i$$

$$\hat{g}_i = \begin{cases} z^G_i M^G , & i = G \\ z^G_i , & i \neq G \end{cases} \quad (2.3)$$

2.3.2.2. Fitting Parameters

These five coupled stochastic differential equations can be represented equivalently by a short-time conditional probability distribution, P, in terms of a Lagrangian, L:

$$P(R\cdot, B\cdot; t+\Delta t | R\cdot, B\cdot; t) = \frac{1}{(2\pi\Delta t)^{5/2}\sigma^{1/2}} \exp(-L\Delta t) \tag{2.4}$$

where σ is the determinant of the inverse of the covariance matrix, the metric matrix of this space, $R\cdot$ represents $\{RT, RBMP\}$, and $B\cdot$ represents $\{BT, BAPC, BTOW\}$. (Here, the prepoint discretization is used, which hides the Riemannian corrections explicit in the midpoint discretized Feynman Lagrangian. Only the latter representation possesses the variational principle useful for arbitrary noise.)

This defines a scalar "dynamic cost function," $C(x, y, z)$,

$$C(x, y, z) = L\Delta t + \frac{5}{2}\ln(2\pi\Delta t) + \frac{1}{2}\ln\sigma \tag{2.5}$$

which can be used with the Very Fast Simulated Re-Annealing (VFSR) algorithm [7], to find the (statistically) best fit of $\{x, y, z\}$ to the data.

The form for the Lagrangian, L, and the determinant of the metric, σ, to be used for the cost function C, is:

$$L = \sum_G \sum_{G'} \frac{(\dot{M}^G - g^G)(\dot{M}^{G'} - g^{G'})}{2g^{GG'}}$$

$$\sigma = \det(g_{GG'}) \,,\; (g_{GG'}) = (g^{GG'})^{-1}$$

$$g^{GG'} = \sum_i \hat{g}_i^G \hat{g}_i^{G'} \tag{2.6}$$

Generated choices for $\{x, y, z\}$ are constrained by empirical (taken from exercises or from computer simulations of these exercises) KVS:

$$g^G(t) \le n^G < \Delta M^G(t) >$$

$$\hat{g}_i^G(t) \le n_i^G [< (\Delta M^G(t))^2 >]^{1/2} \tag{2.7}$$

where n^G and n_i^G are the number of terms in g^G and $\hat{g}_i^G$, resp., and averages, $< \cdot >$, are taken over all time epochs and trajectories of similar scenarios.

If there are competing mathematical forms, then it is advantageous to utilize the path-integral to calculate the long-time evolution of P [11]. Experience has demonstrated that, since P is exponentially sensitive to changes in L, the long-time correlations derived from theory, measured against the empirical data, is a viable and expedient way of rejecting models not in accord with empirical evidence.

Note that the use of the path integral is *a posteriori* to the short-time fitting process, and is a subsidiary physical constraint on the mathematical models to judge their internal soundness and suitability for attempts to extrapolate to other scenarios.

2.3.2.3. Combat Power Scores

After the $\{x, y, z, \}$ are fit to the data, and a mathematical model is selected, another fit can be superimposed to find the effective "combat scores," defined here as the relative contribution of each system to the specific class of scenarios in question. Using a fundamental property of probability distributions, a probability distribution $P_A(q)$ of aggregated variables $q_1 + q_2$ can be obtained from the probability distribution for $P(q_1, q_2)$:

$$P_A(q = q_1 + q_2) = \int dq_1\, dq_2\, P(q_1, q_2)\delta(q - q_1 - q_2) \tag{2.8}$$

where $\delta(\cdot)$ is the Dirac delta function.

Thus, we calculate the aggregated conditional probability

$$P_A(r, b; t + \Delta t | R\cdot, B\cdot; t)$$

$$= \int dRT\ dRBMP\ dBT\ dBAPC\ dBTOW\ \ P(R\cdot, B\cdot; t + \Delta t | R\cdot, B\cdot; t)$$

$$\times \delta(r - w^r_{RT} RT - w^r_{RBMP} RBMP)\ \delta(b - w^b_{RT} BT - w^b_{BAPC} BAPC - w^b_{BTOW} BTOW) \tag{2.9}$$

where the w's represent the desired combat scores. After the $\{x, y, z\}$ have been fitted, the new parameters $\{w\}$ can be fit the data by maximizing the cost function $C'(w)$ using VFSR,

$$C'(w) = -\ln P_A \tag{2.10}$$

Note that the simple linear aggregation by system above can be generalized to nonlinear functions, thereby taking into account synergistic interactions among systems which contribute to overall combat effectiveness.

We will be able to explore the possibility of developing Human Factors Combat Power Scores, since we will be similarly including human-factors variables in such equations, as discussed in Section 3.

2.3.2.4. Algebraic Complexity Yields Simple Intuitive Results

Consider a multivariate system, but with the multivariate variance a general nonlinear function of the variables. The Einstein summation convention helps to compact the equations, whereby repeated indices in factors are to be summed over.

The Itô (prepoint) discretization for a system of stochastic differential equations is defined by

$$\bar{t}_s \in [t_s, t_s + \Delta t]$$

$$M(\bar{t}_s) = M(t_s)$$

$$\dot{M}(\bar{t}_s) = M(t_{s+1}) - M(t_s) \tag{2.11}$$

The stochastic equations are then written as

$$\dot{M}^G = f^G + \hat{g}^G_i \eta^i$$

$$i = 1, \cdots, \Xi$$

$$G = 1, \cdots, \Theta \tag{2.12}$$

The operator ordering (of the $\partial/\partial M^G$ operators) in the Fokker-Planck equation corresponding to this discretization is

$$\frac{\partial P}{\partial t} = VP + \frac{\partial(-g^G P)}{\partial M^G} + \frac{1}{2}\frac{\partial^2(g^{GG'} P)}{\partial M^G \partial M^{G'}}$$

$$g^G = f^G + \frac{1}{2}\hat{g}^{G'}_i \frac{\partial \hat{g}^G_i}{\partial M^{G'}}$$

$$g^{GG'} = \hat{g}^G_i \hat{g}^{G'}_i \tag{2.13}$$

The Lagrangian corresponding to this Fokker-Planck and set of Langevin equations may be written in the Stratonovich (midpoint) representation, corresponding to

$$M(\bar{t}_s) = \frac{1}{2}[M(t_{s+1}) + M(t_s)] \tag{2.14}$$

This discretization can be used to define a Feynman Lagrangian L which possesses a variational principle, and which explicitly portrays the underlying Riemannian geometry induced by the metric tensor $g_{GG'}$, calculated to be the inverse of the covariance matrix.

$$P = \int \cdots \int \underline{D}M \exp(- \sum_{s=0}^{u} \Delta t L_s)$$

$$\underline{D}M = g_{0_+}^{1/2} (2\pi \Delta t)^{-1/2} \prod_{s=1}^{u} g_{s_+}^{1/2} \prod_{G=1}^{\Theta} (2\pi \Delta t)^{-1/2} dM_s^G$$

$$\int dM_s^G \to \sum_{\alpha=1}^{N^G} \Delta M_{\alpha s}^G , M_0^G = M_{t_0}^G , M_{u+1}^G = M_t^G$$

$$L = \frac{1}{2} (\dot{M}^G - h^G) g_{GG'} (\dot{M}^{G'} - h^{G'}) + \frac{1}{2} h^G{}_{;G} + R/6 - V$$

$$[\cdots]_{,G} = \frac{\partial[\cdots]}{\partial M^G}$$

$$h^G = g^G - \frac{1}{2} g^{-1/2} (g^{1/2} g^{GG'})_{,G'}$$

$$g_{GG'} = (g^{GG'})^{-1}$$

$$g_s[M^G(\bar{t}_s), \bar{t}_s] = \det(g_{GG'})_s , g_{s_+} = g_s[M_{s+1}^G, \bar{t}_s]$$

$$h^G{}_{;G} = h^G_{,G} + \Gamma^F_{GF} h^G = g^{-1/2} (g^{1/2} h^G)_{,G}$$

$$\Gamma^F_{JK} \equiv g^{LF} [JK, L] = g^{LF} (g_{JL,K} + g_{KL,J} - g_{JK,L})$$

$$R = g^{JL} R_{JL} = g^{JL} g^{JK} R_{FJKL}$$

$$R_{FJKL} = \frac{1}{2} (g_{FK,JL} - g_{JK,FL} - g_{FL,JK} + g_{JL,FK}) + g_{MN} (\Gamma^M_{FK} \Gamma^N_{JL} - \Gamma^M_{FL} \Gamma^N_{JK}) \tag{2.15}$$

Because of the presence of multiplicative noise, the Langevin system differs in its Itô (prepoint) and Stratonovich (midpoint) discretizations. The midpoint-discretized covariant description, in terms of the Feynman Lagrangian, is defined such that (arbitrary) fluctuations occur about solutions to the Euler-Lagrange variational equations. In contrast, the usual Itô and corresponding Stratonovich discretizations are defined such that the path integral reduces to the Fokker-Planck equation in the weak-noise limit. The term $R/6$ in the Feynman Lagrangian includes a contribution of $R/12$ from the WKB approximation to the same order of $(\Delta t)^{3/2}$ [17].

Now, consider the generalization to many cells. In the absence of any further information about the system, this increases the number of variables, from the set $\{G\}$ to the set $\{G, \nu\}$.

A different prepoint discretization for the same probability distribution P, gives a much simpler algebraic form, but the Lagrangian L' so specified does not satisfy a variational principle useful for moderate to large noise. Still, this prepoint-discretized form has been quite useful in all systems examined thus far, simply requiring a somewhat finer numerical mesh.

It must be emphasized that the output need not be confined to complex algebraic forms or tables of numbers. Because L possesses a variational principle, sets of contour graphs, at different long-time epochs of the path-integral of P over its r and b variables at all intermediate times, give a visually intuitive and accurate decision-aid to view the dynamic evolution of the scenario.

This Lagrangian approach to combat dynamics permits a quantitative assessment of concepts previously only loosely defined.

$$\text{``Momentum''} = \Pi^G = \frac{\partial L}{\partial(\partial M^G/\partial t)}$$

$$``\text{Mass}'' = g_{GG'} = \frac{\partial L}{\partial(\partial M^G/\partial t)\partial(\partial M^{G'}/\partial t)}$$

$$``\text{Force}'' = \frac{\partial L}{\partial M^G}$$

$$``F = ma'':\ \delta L = 0 = \frac{\partial L}{\partial M^G} - \frac{\partial}{\partial t}\frac{\partial L}{\partial(\partial M^G/\partial t)} \tag{2.16}$$

where M^G are the variables and L is the Lagrangian. These relationships are derived and are valid at each temporal-spatial point x of $M^G(x, t)$. Reduction to other math-physics modeling can be achieved after fitting realistic exercise and/or simulation data.

These physical entities provide another form of intuitive, but quantitatively precise, presentation of these analyses.

2.3.3. Numerical Methodology and Results

Recently, two major computer codes have been developed, which are key tools for use of this approach to mathematically model combat data.

The first code, Very fast Re-Annealing (VFSR) [7], fits short-time probability distributions to empirical data, using a most-likelihood technique on the Lagrangian. An algorithm of very fast simulated re-annealing has been developed to fit empirical data to a theoretical cost function over a D-dimensional parameter space [7], adapting for varying sensitivities of parameters during the fit. The annealing schedule for the "temperatures" (artificial fluctuation parameters) T_i decrease exponentially in "time" (cycle-number of iterative process) k, i.e., $T_i = T_{i0} \exp(-c_i k^{1/D})$.

Heuristic arguments have been developed to demonstrate that this algorithm is faster than the fast Cauchy annealing [36], $T_i = T_0/k$, and much faster than Boltzmann annealing [37], $T_i = T_0/\ln k$. To be more specific, the kth estimate of parameter α^i,

$$\alpha^i_k \in [A_i, B_i] \tag{2.17}$$

is used with the random variable x^i to get the $k+1$th estimate,

$$\alpha^i_{k+1} = \alpha^i_k + x^i(B_i - A_i)$$

$$x^i \in [-1, 1] \tag{2.18}$$

Define the generating function

$$g_T(x) = \prod_{i=1}^{D} \frac{1}{2\ln(1 + 1/T_i)(|x^i| + T_i)} \equiv \prod_{i=1}^{D} g^i_T(x^i)$$

$$T_i = T_{i0} \exp(-c_i k^{1/D}) \tag{2.19}$$

The cost-functions $\underline{L}$ we are exploring are of the form

$$h(M;\alpha) = \exp(-\underline{L}/T)$$

$$\underline{L} = L\Delta t + \frac{1}{2}\ln(2\pi\Delta t g_t^2) \tag{2.20}$$

where L is a Lagrangian with dynamic variables $M(t)$, and parameter-coefficients α to be fit to data. g_t is the determinant of the metric. Note that the use of $\underline{L}$ is *not* equivalent to doing a simple least squares fit on $M(t + \Delta t)$.

The second code develops the long-time probability distribution from the Lagrangian fitted by the first code. A robust and accurate histogram path-integral algorithm to calculate the long-time probability distribution has been developed to handle nonlinear Lagrangians [38-40], including a two-variable code for additive and multiplicative cases. We are presently working to create a code to process several variables, blending VFSR [7] with more traditional Monte Carlo path-integral methods.

2.4. MODELING OF NTC

2.4.1. Fits to Data

The ‘‘kills’’ attrition data from NTC and our JANUS(T)/NTC simulation at once looks strikingly similar during the force-on-force part of the combat. (See Fig. 2.1.) Note that we are fitting (only half) the middle part of the engagement, where the slope of attrition is very steep (and almost linear on the given scale), i.e., the ‘‘force on force’’ part of the engagement. The second half of the data must be predicited by our models.

Figure 2.1
NTC Versus JANUS(T)

From the single NTC trajectory qualified to date, 7 five-minute intervals in the middle of the battle were selected. From six JANUS(T) runs, similar force-on-force time epochs were identified, for a total of 42 data points. In the following fits, r represents Red tanks, and b represents Blue tanks.

Fitting NTC data to an additive noise model, a cost function of 2.08 gave:

$$\dot{r} = -2.49 \times 10^{-5} b - 4.97 \times 10^{-4} br + 0.320 \eta_r$$

$$\dot{b} = -2.28 \times 10^{-3} r - 3.23 \times 10^{-4} rb + 0.303 \eta_b \quad (2.21)$$

Fitting NTC data to a multiplicative noise model, a cost function of 2.16 gave:

$$\dot{r} = -5.69 \times 10^{-5} b - 4.70 \times 10^{-4} br + 1.06 \times 10^{-2} (1 + r) \eta_r$$

$$\dot{b} = -5.70 \times 10^{-4} r - 4.17 \times 10^{-4} rb + 1.73 \times 10^{-2} (1 + b) \eta_b \quad (2.22)$$

Fitting JANUS(T) data to an additive noise model, a cost function of 3.53 gave:

$$\dot{r} = -2.15 \times 10^{-5} b - 5.13 \times 10^{-4} br + 0.530 \eta_r$$

$$\dot{b} = -5.65 \times 10^{-3} r - 3.98 \times 10^{-4} rb + 0.784 \eta_b \quad (2.23)$$

Fitting JANUS(T) data to a multiplicative noise model, a cost function of 3.42 gave:

$$\dot{r} = -2.81 \times 10^{-4} b - 5.04 \times 10^{-4} br + 1.58 \times 10^{-2} (1 + r) \eta_r$$

$$\dot{b} = -3.90 \times 10^{-3} r - 5.04 \times 10^{-4} rb + 3.58 \times 10^{-2} (1 + b) \eta_b \quad (2.24)$$

This comparison illustrates that two different models about equally fit the short-time distribution. The multiplicative noise model shows that about a factor of 100 of the noise might be ‘‘divided out,’’ or understood in terms of the physical log-normal mechanism.

In order to discern which model best fits the data, we turn to the path-integral calculation of the long-time distribution, to see which model best follows the actual data.

Fig. 2.2 presents the long-time probability of finding values of these forces. In general, the probability will be a highly nonlinear algebraic function, and there will be multiple peaks and valleys.

Figure 2.2
Path-Integral Calculation of Long-Time Distribution

Figs. 2.3 and 2.4 give the means and variances of tank attrition from the JANUS(T) and NTC databases. Since we presently have only one NTC mission qualified, the variance of deviation from the mean is not really meaningful; it is given only to illustrate our approach which will be applied to more

NTC missions as they are qualified and aggregated. Note that only the Blue JANUS(T) variances of the additive noise model are consistent with the NTC data.

Figure 2.3
Attrition Means

Figure 2.4
Attrition Variances

2.4.2. Discussion of Study

Data from 35 to 70 minutes was used for the short-time fit. The path integral code was used to calculate the long-time evolution of this fitted short-time (5-minute) distribution from 35 minutes to beyond 70 minutes. This serves to compare long-time correlations in the mathematical model versus the data, and to help judge extrapolation past the data used for the short-time fits. More work is required to find a better (or best?) algebraic form. The resulting form is required for input into higher echelon models. As more NTC data becomes available (other NTC missions are in the process of being qualified, wargamed and analyzed), we will be able to judge the best models with respect to how well they extrapolate across slightly different combat missions.

2.4.3. Chaos or Noise?

Given the context of current studies in complex nonlinear systems, the question can be asked: What if combat has chaotic mechanisms that overshadow the above stochastic considerations? The real issue is whether the scatter in data can be distinguished between being due to noise or chaos. Several studies have been proposed with regard to comparing chaos to simple filtered (colored) noise [J. Theiler, private communication] [41, 42].

The combat analysis was possible only because now we had recent data on combat exercises from the National Training Center (NTC) of sufficient temporal density to attempt dynamical mathematical modeling. The criteria used to (not) determine chaos in this dynamical system is the nature of propagation of uncertainty, i.e., the variance. For example, following by-now standard arguments [J. Yorke, seminar and private communication], propagation of uncertainty may be considered as (a) diminishing, (b) increasing additively, (c) or increasing multiplicatively. An example of (a) is the evolution of a system to an attractor, e.g., a book dropped onto the floor from various heights reaches the same point no matter what the spread in initial conditions. An example of (b) is the propagation of error in a clock, a cyclic system. Examples of (c) are chaotic systems, of which very few real systems have been shown to belong. An example of (c) is the scattering of a particle in a box whose center contains a sphere boundary: When a spread of initial conditions is considered for the particle to scatter from the sphere, when its trajectories are aligned to strike the sphere at a distance from its center greater that the diameter, the spread in scattering is a factor of about three greater than the initial spread.

In our analysis of NTC data, we were able to fit the short-time attrition epochs (determined to be about 5 minutes from mesh considerations determined by the nature of the Lagrangian) with short-time nonlinear Gaussian-Markovian probability distributions with a resolution comparable to the spread in data. When we did a long-time path-integral from some point (spread) at the beginning of the battle, we found that we could readily find a form of the Lagrangian that made physical sense and that also fit the multivariate variances as well as the means at each point in time of the rest of the combat interval. I.e., there was not any degree of sensitivity to initial conditions that prevented us from ‘‘predicting’’ the long time means and variances of the system. Of course, since the system is dissipative, there is a strong tendency for all moments to diminish in time, but in fact this combat is of sufficiently modest duration (typically 1 to 2 hours) that variances do increase somewhat during the middle of the battle.

In summary, this battalion-regiment scale of battle does not seem to possess chaos. Of course, some other combat conditions might show some elements of chaos in some spatial-temporal domain, and then the resolution of the analysis would determine the influence of that chaos on that combat scenario.

2.4.4. Conclusion

We have demonstrated proofs of principle, that battalion-level combat exercises can be well represented by the computer simulation JANUS(T), and that modern methods of nonlinear nonequilibrium statistical mechanics can well model these systems. Since only relatively simple drifts and diffusions were required, it might be possible to ‘‘absorb’’ other important variables (C^3, human factors, logistics, etc.) into more nonlinear mathematical forms. Otherwise, this battalion-level model should be supplemented with a ‘‘tree’’ of branches corresponding to estimated values of these variables. This is discussed further in Section 3.

For this new project, it is important to note that the requirement of fitting a dynamical theoretical model across the (half-)domain of force-on-force battle, at appropriately short time intervals, is quite a severe constraint on the model. Even data from just one NTC scenario seemed sufficient to pin down our theoretical model.

3. Mathematical Modeling of Human Factors in Teleoperated Vehicles

3.1. INTRODUCTION

Military C3 (Command, Control and Communications), while supported by an imposing array of sensors, computers, displays, communications and weapons is in its essence a human decision making activity. A trained individual has a unique talent for recognizing changes in situation, and for allocating resources appropriately in a dynamically varying encounter which is subject to significant uncertainty. Such skills are utilized, for example, in an application in which a person must distinguish the highest priority target in a cluster of like objects in an environment permeated with a high level of visual clutter. In this case the commander brings a semblance of order to observations of the motion of interacting groups of hostile and friendly vehicles, and on the basis of his understanding of the implications of the perceived events, he can make rational judgments on suitable strategies. Indeed, the ability of a human to employ powers of analogical reasoning and problem restructuring in response to sudden and unexpected events contrasts sharply with algorithmic approaches to decision making problem [43].

A full exploitation of this human capability is sometimes difficult to achieve. An environment conducive to good decision making is frequently lacking at precisely the time it is most necessary. To decrease the commander's stress level some of the more primitive human roles may be assigned to an autonomous subsystem. For example, an algorithmic surrogate could perform certain observation and decision making functions without requiring direct human action. Unfortunately, such autonomous systems have not fulfilled much of their initial promise; e.g. the implications of cluttered observations of the motions of a multiple-vehicle, dynamic encounter are unfathomable by algorithms derived on the basis of current technology. At present, only the most modest and precisely focused tasks are capably dealt with autonomously.

There are some relevant differences between the milieu as it appears to a commander, and what can be assimilated by an algorithmic counterpart. To clearly distinguish relevant human capabilities, it is important to study the interpretive capability of a commander as a function of the nominal descriptors of the decision making environment, and determine how this relates to his performance in a dynamic encounter. To provide a description of human behavior that will aid in defining the appropriate human role in the system architecture, it is advantageous to have a behavioral model which is both relatively simple and compatible with the models of the other components of the encounter. Analytical models of human response have a long history, with careful development beginning in the 1940's. More recently, as the tasks assigned to the human have become more multifarious, increasingly sophisticated models have been required. In [44] Johannsen and Rouse proposed a framework within which human activities could be organized. Their hierarchical perspective is amenable to a quantitative computer-like interpretation of human functions, but at the same time accounts for higher level psychological and intellectual activities such as reflection and planning. At the lowest level, the commander behaves in an essentially automatic way. Indeed, in highly trained commanders, proper behaviors, once learned, become reflexive and are probably performed at the level of the cerebellum. Johannsen and Rouse point out that the events which elicit these activities tend to occur relatively frequently and the response becomes instinctive. Rasmussen [45] continued the trend toward a hierarchical representation of human control and decision making behaviors and provided a taxonomy of human behavior patterns.

When concern centers on such composite problems as tracking of a dynamic target in clutter, the Optimal Control Model (OCM) of Baron, Kleinman and Levison has proven quite useful (see [46] for a clear description of this approach along with numerous references). Normative-descriptive models of which the OCM is a notable example have as their rationale the fact that "the motivated expert decision-makers strive for optimality but are constrained from achieving it by inherent human perceptual limitations and cognitive biases." [47] When applied to situations in which actions must be taken in response to an evolving encounter, the OCM and its more recent counterparts have been phrased within the Stimulus/Hypothesis/Options/Response (SHOR) paradigm of human decision making [48].

In [49] Wohl, et. al. discuss some of the fundamental issues which must be addressed in the formulation of the Stimulus and Hypothesis Evaluation portion of the supervisory model. Paraphrasing these themes in the current context, as a commander surveys a region of the battlefield, various constellations of objects may be encountered. There may be important targets of various types, decoys or target like objects of little significance, open areas containing nothing of interest, etc. The commander classifies the current mode of the encounter in terms of a set of a priori hypotheses, and selects the ostensible mode of evolution on the basis of his observations. If this decision is made contingent on the motion patterns of the observed objects, then the hypothesis evaluation block in the response model becomes a bank of Kalman filters tuned to the various dynamic hypotheses, along with a suitable combination of the outputs to generate the conditional likelihoods of the various hypotheses.

In [50] an alternative model (Stimulus/Hypothesis Evaluation Model, SHEM) of the human response is proposed in a tactical application. The hypotheses which give structure to the encounter are more clearly distinguished by their panoramic features than they are by their local motion attributes. The human acts as an explicator of ambiguous observations. For example, a tank (high priority) may maneuver in concert with other vehicles of less worth. The commander will tend to identify the relevant object in the field of view on the basis of its visual signature rather than by its motion pattern. Clearly, motion cues and visual aspect are complementary stimuli. In the application which gave rise to [50], however, attention is centered on ranges at which the extended properties of an object provide a superior indication of the realized hypothesis than does the motion.

For these types of applications, a recognition model having peculiar properties is required. It has been observed that the human explicator has two noticeable cognitive proclivities [51]:

RECENCY: Subjects consistently "overweigh" recent information with respect to conventional normative models.
ANCHORING: Subjects consistently anchor on prior knowledge.

Although the cursory descriptions of recency and anchoring make them appear to be mutually exclusive behaviors, more reflection indicates that they are two distinct aspects of a multifaceted behavior pattern. When a person is convinced that he has identified the status of a situation, he will tend to under appreciate the value of new data. This is an important trait because the brief occurrence of inconsistent information will cause little vacillation. An assured person is unwavering or "anchored" in his belief.

Alternatively, when a person is exposed to an ambiguous data stream of reasonable length, he is much more open to a modification in his view of the condition of the environment. "Recency" is manifested by a high valuation placed upon more recent observations when a person is unsure of the current status of his surroundings. Thus, recency and anchoring are not contradictory behavior patterns, but are instead descriptions of the human response characteristics in different regions of his knowledge space.

In this paper, the relationship between stimulus and hypothesis evaluation is described within the context of tasks that are encountered in a typical C3 application. The explicator portion of the SHEM is a nonlinear stochastic differential equation which quantifies the commander's uncertainty when attempting to distinguish the relevant features in a changing encounter. Proper understanding of the impact of this uncertainty on overall system performance is essential if the human presence is to be best utilized. The overall C3 environment is characterized by three descriptors: the frequency of changes in condition; the distinguishabilty of these changes; and the level of clutter in the observation. The next section quantifies the dependence of the commander's performance on these factors. A distinctive human skill is the ability to recognize novel situations and adapt to them. The pliancy of the model in this hypothesis generation role has also been explored. The degree to which the SHEM can be anthropomorphized depends upon how well its response can be made to mimic that of a human in a similar environment. A test comparing the response of the model with that of a human subject suggests that "human" indecisiveness is captured by the model.

3.2. HUMAN RESPONSE MODELLING

Since one of the primary functions of a C3 system is proper allocation of scarce resources, it is important to model the commander's proficiency in situation assessment. The ability to infer the mode of evolution of an encounter requires a panoramic view, and a faculty for placing the observations within a well defined pattern. The human role becomes preeminent when the encounter involves sudden and unpredictable changes in the operational environment. An appearance of a cluster of targets, a change in demeanor of the cluster in a manner which indicates a threat to friendly forces are possible events that the commander may experience and to which he must respond.

In many C3 systems, the data transmitted to the supervisor has a space-time partition into a temporal data flow, and a sequence of "spatial frames" which are data clusters associated with panoramic features of the encounter at a fixed time. Often the relevant dynamic hypotheses are more clearly differentiated by their global features than they are by any peculiar local motions. Supervisory misperceptions arise when the spatial frames admit multiple interpretations, and there is a need for descriptive models which emulate the process of human recognition of patterns. It has been suggested that a supervisor infers system status by assessing the resemblance of his observations to prototypical forms called "noetic descriptive schema." [52] A human expert is frequently unable to describe precisely the norm for assessing conformity to these perceptual templates. Because the observed modalities of human response are so varied, most detailed investigations of human response have sought to minimize the influence of situational misperceptions.

In the referenced analytical procedures for modeling human response, behavior is determined implicitly by a specification of the task to be accomplished, and the environment in which this must be done. This approach leads to a parametric model; e.g., the OCM, and careful analysis of the sample response of a representative group of subjects yields the data upon which a specialization of the model can be made. Direct utilization of this approach in a C3 environment is impeded by the convoluted form of the equations of the relevant battle dynamics. This issue is articulated clearly by IFW in [1] in which a model of the generic form

$$dx_p = f(x_p, u_p, t)dt + g(x_p, t)dw \tag{3.1}$$

is proposed. The base state x_p is an aggregate vector combining the local attributes of both the Blue and Red forces and $\{w_t\}$ is a vector Brownian motion chosen to model the uncertainty, both continuous and discontinuous, which exists in the evolution of the encounter. The action variable u_p is not explicit in [2], but must be introduced to permit a supervisor to direct the flow of the encounter.

This general C3 model is intractable, and the class of decision strategies which can reasonably be considered is quite small. Even when the equations are simplified considerably, the supervisory decisions are essentially restricted to engagement and disengagement thresholds. Further, the use of a continuous exogenous process $\{w_t\}$ in (3.1) obscures the anomalous influence of discrete disturbances. The specific representation of the exogenous process is particularly relevant here because the commander is "mostly concerned with the unexpected, the unusual and the nonroutine aspects of system control." [53] These unexpected, unusual and nonroutine events are most clearly distinguished by discrete changes in the mode of battle occurring at unpredictable times.

To gain insight into the supervisor's perception of the encounter, a more precise analytical framework is useful. Denote the underlying probability space on which the system disturbances are defined by $(\Omega, \underline{\Psi}, P)$, and let $\{\underline{\Psi}_t\}$ be a filtration on this space. IFW's model can be rewritten to make clear its dependence on alternative modal hypotheses:

$$dx_p = f(x_p, u_p, r_t)dt + g(x_p, u_p)dw_t$$

$$x_{p0} = C \tag{3.2}$$

where $\{r_t\}$ is a modal indicator used to denote the exogenous conditions which determine the manner in which the base states evolve. In keeping with the referenced works, $\{r_t\}$ is assumed to move on a set S with s elements. If $r_t = i$, let $\phi_t = e_i$, denoting the unit vector in the i'th direction in R^s by e_i. Then the modal state $\{\phi_t\}$ evolves on R^s, and indexes the dynamic character of the base states x_p. Equation (3.2) can be phrased in terms of $\{\phi_t\}$ in an obvious way.

The dynamical description of the encounter is not complete until the behavior of $\{\phi_t\}$ has been specified. The simplest model which provides variability in the modal state is that in which $\{r_t\}$ is a $\underline{\Psi}_t$-Markov process. In this event, the local dynamics of $\{r_t\}$ are described by the transition matrix $Q = [q_{ij}]$

$$\Pr(r_{t+\delta} = j | r_t = i) = \begin{cases} 1 + q_{ii}\delta + o(\delta);\ i = j \\ q_{ij}\delta + o(\delta);\ i \neq j \end{cases} \tag{3.3}$$

The supervisor's perception of the encounter is based upon data provided by a communication network. The observation sequence, $\{\Theta_t\}$, generates a subfiltration $\{\underline{\Theta}_t\}$ of $\{\underline{\Psi}_t\}$. It will be assumed that

$\{\Theta_t\}$ has the decomposition paralleling that of the state: $\Theta_t = (z_t, y_t)$ where z_t is a measurement of the base states and y_t is a direct modal measurement. This partition is evocative, not exclusive. The base and modal states are coupled in (3.2). Hence a measurement of either is to some degree a measurement of the other. Still, the quality of the observation links may be such that this identification is natural.

Suppose that measurement of the base state has the conventional form

$$dz_t = Hx_t dt + dv_t \tag{3.4}$$

where $\{v_t\}$ is $\underline{\Psi}_t$-Brownian motion with intensity $R > 0$, and independent of $\{w_t\}$. The modal variable manifests itself more subtly in a spatiotemporal sensory signature. In [52] these signatures are described as "behavioral, structural and relational (bsr) patterns." To be specific, suppose that each mode has a particular signature h_i. A cluttered measurement, $\{y_t\}$, of the signature associated with current modal state is received by the supervisor. This measurement will be represented by the stochastic differential equation

$$dy_t = h^t \phi_t dt + dn_t \tag{3.5}$$

where h is the indicated s-vector, and $\{n_t\}$ is $\underline{\Psi}_t$-Brownian motion independent of v and w.

Equation (3.5) is a suppositional relation which associates the noetic form (ϕ_t) to the bsr pattern h. The capacity and fidelity of the communication network is reflected in (3.5) along with the exogenous phenomena in the encounter by the distinguishability of the modal hypotheses ($\|h_i - h_j\|$), and the wideband clutter $\{n_t\}$. The former has a static character, while the latter represents the indigenous noise associated with the measurement and transmission of information. The effect of external clutter, communication system fidelity, and characteristic misclassifications of the post-processing algorithms are aggregated synergistically in this model. On the basis of $\{\Theta_t\}$ and more specifically $\{y_t\}$, the decision maker infers the likelihood of the various modal alternatives. Let $\{\underline{Y}_t\}$ be the natural filtration associated with $\{y_t\}$. Denote the conditional expectation with respect to $\{\underline{Y}_t\}$ or $\{\underline{\Theta}_t\}$ by "$\hat{}$" as appropriate. Then the commander's conception of the relative probabilities of different mode is given by the "explicator state" $\{\hat{\phi}_t\}$

$$\begin{aligned}\{\hat{\phi}_t\} &= \mathrm{E}\{\phi_t | \underline{Y}_t\} \\ &= \Pr[(\phi_t = e_i | \underline{Y}_t)]\end{aligned} \tag{3.6}$$

The equations of evolution of $\{\hat{\phi}_t\}$ are given in [54]. This process was used to describe the cognitive response of an operator directing a teleoperated vehicle in a multitask environment:

$$d\hat{\phi}_t = Q^t \hat{\phi}_t dt + (\mathrm{diag}\, h - \hat{\phi}_t^t h I)\hat{\phi}_t d\nu_\phi \tag{3.7}$$

where I is the identity matrix and $\{\nu_\phi\}$ is the innovations process

$$d\nu_\phi = dy_t - h^t \hat{\phi}_t dt \tag{3.8}$$

The analytical description of the supervisor's environment is completed by the specification of a performance measure which permits the comparison of alternative action policies. A category of such measures which has been found to be useful in numerous studies is given by J_0 where

$$J_t(\{u_p\}) = \mathrm{E}\{ \int_t^T c(x_p, u_p, r) d\tau | \underline{\Theta}_t \} \tag{3.9}$$

The $\underline{\Theta}_t$-adapted process which minimizes $J_0(\{u_p\})$, and satisfies appropriate additional system level constraints is termed the optimal supervisory policy for this problem.

The supervisory model is given by the mapping from observation to action. To find this mapping, the optimization problem given above must be solved explicitly. Such problems are of the "dual control" genre; so called because the actuating signal must probe the system to determine the current value of $\{\phi_t\}$ while simultaneously performing the tasks implicit in the performance functional. The study of this class of problems has a long history, and no generally acceptable solution procedure has yet been advanced. In principle, the equation for $\{\phi_t\}$ can be appended to (3.2) to form the full state equation for the system, and the verification theorem of dynamic programming [55] can be used to give necessary conditions for optimality. Unfortunately, the resulting equations are intractable in most cases. To provide an explicit model, the encounter description must be simplified. Suppose, therefore, that for each of the modal values that there is a nominal base state and action; i.e. there is a set of nominal pairs indexed by r; $\{x_n(r), u_n(r); r \in S\}$. For convenience the nominal paths are assumed to be constants, but this is not essential.

The supervisor will attempt to maintain the state and action variables close to their nominal values. Define the perturbation variables $\{x_t, u_t\}$ by

$$x_{pt} = x_n(\phi_t) + x_t$$

$$u_{pt} = u_n(\phi_t) + u_t \tag{3.10}$$

Then, if $\{x_t, u_t\}$ is small and f, g in (3.1) smooth, the perturbation dynamics for IFW's model can be approximated by a set of linear equations

$$dx_t = (F_i x_t + G_i u_t)dt + gdw_t; \text{if } \phi_t \equiv e_i \tag{3.11}$$

where (F_i, G_i) are gradients evaluated about the associated nominal.

While (3.11) provides a good system model during quiescent operation, when $\{\phi_t\}$ changes, the situation changes quickly. From (3.7) if $\{\phi_t\}$ makes an $i \to j$ transition at time s, then $x_s = x_{s^-} + \delta(j, i)$ where $\delta(j, i) = x_n(i) - x_n(j)$. This can be incorporated into (3.11) to yield

$$dx_t = (F_i x_t + G_i u_t)dt + gdw_t + \rho^t d\phi_t; \text{if } \phi_t = e_i \tag{3.12}$$

The supervisory model is given by the mapping from $\{\underline{\Theta}_t\}$ to u_p. This has two parts as indicated in (3.10). The construction of $u_n(\phi_t)$ is a topic of current study. Here, the focus will be on the properties of $\{u_t\}$, and its dependence upon the disjoined observations. A problem of this type was studied in [56], again in a tactical application. Suppose that the coefficient matrices in (3.12) are independent of m, ϕ and that $P_t = \mathrm{Var}\{x_t | \underline{\Theta}_t\} = \mathrm{Var}\{x_t | Y_t\}$ [50]. Then the conditional mean of the base state satisfies the equation

$$d\hat{x}_t = (F\hat{x}_t + Gu_t + \rho^t Q^t \hat{\phi}_t)dt + P_t H^t R^{-1} d\nu_x \tag{3.13}$$

where $\nu_x = z_t - \hat{z}_t$. Let the criterion function be quadratic,

$$J_t(\{u_p\}) = \mathrm{E}\{ \int_t^T (\hat{x}^t M \hat{x} + u^t N u) d\tau \} \tag{3.14}$$

It is shown in [3] that the Local Action Response Equation (LARE) of the supervisor is given by

$$u = -N^{-1} G^t (\Sigma_t \hat{x} + \Upsilon_t) \tag{3.15}$$

where

$$\dot{\Sigma}_t = -M - \Sigma F - F^t \Sigma + \Sigma G N^{-1} G^t \Sigma$$

$$\Sigma_T = 0 \tag{3.16}$$

and

$$\hat{\Upsilon}_t \equiv \varnothing_t \hat{\phi}_t \tag{3.17}$$

with

$$\dot{\varnothing}_s = \Sigma \rho^t Q^t + (F^{t-} \Sigma G N^{-1} G^t) \varnothing_s + \varnothing_s Q^t; \; s \ge t$$

$$\varnothing_T = 0 \tag{3.18}$$

The supervisory equation has an interesting decomposition into a portion related to the estimate of the base state $(-N^{-1} G^t \hat{\Sigma}_t \hat{x})$, and a portion influenced by the modal uncertainty $(-N^{-1} G^t \hat{\Upsilon}_t)$. The former is closely related to the task specification in so far as the gain accorded the estimate of the base state error is not dependent upon the characteristics of the observation. The intensity of the disturbances in this link does manifest itself in the dynamics of the base state estimate through the factor R. Note that the error variance $\{P_t\}$ is a random process which is dependent upon the modal evolution.

The model of the commander's interpretive process, (3.7), is an Ito equation, and considerable care must be exercised in its interpretation. The effect that the rate of change in the encounter, the fear discernibility, and the level of exogenous clutter have on hypothesis evaluation have been exhibited in the sample behavior of the SHEM. For different sample functions of the observation noise, the commander's perception of the scenario features will change. As a consequence, $\{\phi_t\}$ is a random process even for a predetermined scenario $\{\phi_t\}$. Thus one would not expect the model to reproduce a specific human response any more than a human would be unaffected by the realization of the exogenous visual clutter, or indeed that two humans would respond identically to the same stimuli. With this caveat, (3.7) does indicate an appropriate indecisiveness in identifying feature changes.

3.3. HUMAN RESPONSE CHARACTERISTICS

The adequacy of the normative-descriptive model of a human decision maker must be established empirically. The previous section has indicated in some detail the performance attributes of the input-output model of the commandeer. In the SHOR paradigm the decision maker responds to new data by modifying the likelihoods of the various hypotheses which delineate the encounter. The SHEM provides an algorithmic description of the precise way in which this is done.

The hypothesis evaluation portion of the SHEM is difficult to verify because this appraisal is an internal activity of the decision maker, and is normally reflected only indirectly by his actions. Nevertheless, to better understand the limitations of the SHEM, a scene recognition task of the type to be encountered in an application involving a teleoperated vehicle (TOV) was given to a representative operator. The experimental facilities were rudimentary, and the operational environment of the TOV operator could not be reproduced. The experimental protocol can be described as follows. A set of photographs were taken at the Camp Pendleton Marine Base in California. They were appropriately juxtaposed to from a panoramic view of the terrain as seen from a fixed location. The scene consisted of rather open terrain containing scrub brush along with a scattering of discrete objects at various ranges. The actual object set contained jeeps, Land Vehicle, Tracked (LVTs), out buildings, tanks, and various natural structures similar in appearance to the foregoing. The vehicles had different aspect angles with respect to the viewer and had a variety of visual appearances.

To test the dynamic response of a decision maker, a single category of objects was made that of primary concern. In the experiment, the operator was asked to identify the presence of a tank in a changing scene, and to consider all other events as being inconsequential. At one level then the operator could be viewed as comparing a simple hypothesis (tank present) with a single alternative (tank absent). Actually both hypotheses are composite, with the latter containing many well defined subsidiary hypotheses. Hence, this simple structure with $S = \{1, 2\}$ is not sufficiently rich to capture the scenario as the decision maker perceives it.

There was considerable visual ambiguity since other object classes share many features with a tank when viewed at a distance. Indeed, anything with an angular shape could be confused with the primary target class at first glance. It was necessary, therefore, to introduce a third operative hypothesis (secondary object present) to account for the decoys. Surprisingly, given the intricacy of the scene, further additions to S were not found to be necessary. It should be noted that the SHEM permits augmentation to the modal set in a direct manner if the actions of the decision maker warrant such an increase in dimension.

To inject scene dynamics into what is fundamentally a static encounter, a movable camera was made to pan a horizontal slice in the picture at a fixed angular rate. A local image was displayed on a monitor for the operator to view. Thus, static objects appeared in the monitor at random times with random clarity but with duration fixed by the linear dimension. The scan rate thus determined the time scale of changes in the events to which the operator responded. A frequent modal change is achieved at a high scan rate. At low scan rates, the operator has more time to contemplate and distinguish the relevant objects from clutter.

The subject was given a joy stick with one degree of freedom, and was asked to position the rod to indicate her level of confidence that a target was contained in the image displayed on the monitor. The output voltage from the joy stick was proportional to the angular position with the lowest voltage representing certainty that there was not a target within the field of view, and maximum voltage representing certainty that a tank was displayed on the monitor. An intermediate position would be appropriate when the scene is ambiguous.

Figure 3.1 displays a sample function of the subject's response. As mentioned earlier, the event sequence can be viewed as consisting of an alternation between distinct modal hypotheses. The actual time intervals during which a target is on the monitor are indicated in the figure by the dotted rectangles. Let the event that a tank is being observed be denoted by $r_t = 1$. Decoys are similarly distinguished by the dashed rectangles with the corresponding event denoted by $r_t = 2$. The remainder of the interval consisted of open fields and unstructured clutter, and this is denoted by $r_t = 3$.

Figure 3.1
Operator response to targets (dotted) and decoys (dashed)

From a cursory review of the picture, the broad outlines of the decision maker's behavioral peculiarities shown in Figure 3.1 are easily predicted. During sojourns in regions of relatively open terrain, there is little operator confusion. The output of the joy stick should be essentially zero during such phases.

As the picture was scanned, there were regions in which isolated and distinguishable objects were encountered, and the subject attempted to identify them. The scanning direction is such that an object enters the subject's visual field from the right of the screen. When an object appears on the monitor, the operator will initially be uncertain as to the object classification. If the object has a distinct, angular shape that separates it from the more diffuse background, the operator will realize that a modal change has taken place but may delay making a confident identification. The subject indicates this confusion by moving the rod to an intermediate position which is roughly proportional to the probability that the object is in the primary category. In an object sparse region, the operator will concentrate on an unusual shape as it moves across the screen, and make more definitive judgments on the proper classification. Some of the things in the scene are quite deceptive. Careful study is required to distinguish these decoys from targets. The presence of these "near targets" produces significant false alarms.

In another region in the picture, a group of similar objects are in close proximity. While there is never a situation in which more than one object is within the field of view of the camera, when objects occur in rapid succession, the observer never has the time to focus clearly on the object at hand. Instead, the decision maker becomes preoccupied with the sequence itself, and lapses into an unsettled state in which the objects are not clearly differentiated.

In Figure 3.1 the operator response is shown along with the regions of targets and decoys. The operator response is related to $\{\phi_1\}$ in the notation of (3.7). The anticipated peculiarities of the decision maker's response characteristics appear in the sample function. Open areas are clearly recognized as such for the most part, although some natural objects do cause confusion at certain light angles. What is more interesting is the response of the operator to targets and putative targets. Predictably, the operator has much more difficulty in differentiating the hypotheses when the objects appear more frequently than when they are separated by extended intervals.

A simulated response of the hypothesis evaluation portion of the SHEM is shown in Figure 3.2. The most notable difference between the simulated and the actual response is that the simulated sample function is locally much more volatile than is the human. This is not surprising. Equation (3.7) is an Ito equation, and solutions to such equations are quite irregular on any short time scale. The decision maker's evaluation of a situation is probably not nearly so changeable, and even if it were, the physiological lags which occur in translating a mental view of a scene into a shaft position would smooth the volatility and preclude its measurement. Furthermore, a pointing system under the direction of the operator has a low pass character as well. Hence, the local volatility is not an important issue in human modeling.

Figure 3.2
Sample function for the test scenario

With this caveat, the response of the SHEM has the "human" peculiarities that were predicted and measured. This similarity of response manifests itself clearly in the object rich portion of the scan. The rapid modal variation is represented in the SHEM by a Q matrix with high modal transition rates. As the modal variation becomes more frequent, the response of the SHEM becomes more indecisive.

3.4. CONCLUSIONS

A human decision maker plays a central role in many system architectures. Unfortunately it is difficult to delineate his characteristics in an analytically tractable manner. He is capable of so many different behaviors that it is hard to capture his persona in an analytical form. This paper presents a model of one portion of his response characteristics, and indicates the capability and versatility of the hypothesis evaluation portion of the SHEM. Some very human peculiarities have been observed in tests utilizing the model. The indecisiveness and uncertainty under which the decision maker must perform his assigned tasks manifest themselves in the simulation study. As has been pointed out, the proper response of the SHEM is dependent on the correct selection of the model parameters. The simulations indicate that considerable freedom exists in shaping the response characteristics. Indecisiveness in a rapidly changing scenario can be produced by increasing the size of the elements in the Q matrix. If two or more hypotheses are difficult to differentiate, the associated elements of the h vector should be made close.

Situation evaluation is only one component of the commander's function. The complete model requires that a characterization of the Options/Response. This is a topic of current study.

FIGURES

Figure 2.1. NTC Versus JANUS(T).

Attrition (‘‘kills’’) data for the 5 systems in Eq. (2.3) is illustrated for an NTC mission (upper left box) and for three JANUS(T) runs using the NTC-qualified database.

Figure 2.2. Path-Integral Calculation of Long-Time Distribution.

In this plot, the horizontal axes represent Red and Blue forces. For this JANUS(T) additive noise case, two time slices are superimposed. Taking the initial onset of the force-on-force part of the engagement as 35 minutes on the JANUS(T) clock, these peaks represent 50 and 100 minutes. Reflecting boundary conditions are taken at the beginning values of Red and Blue Tanks. Exit boundary conditions are taken at the other two surfaces.

Figure 2.3. Attrition Means.

The left two boxes represent Blue; the right two boxes represent Red. The LHS box of each pair represents JANUS(T); the RHS box represents NTC. Solid lines are the additive noise model; dotted lines are the multiplicative noise model. Small circles in the Means’ boxes are empirical data.

Figure 2.4. Attrition Variances.

Figure 3.1. Operator response to targets (dotted) and decoys (dashed).

Figure 3.2. Sample function for the test scenario.

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