Statistical mechanics of neocortical interactions: EEG correlates of reaction times

Lester Ingber

Lester Ingber Research
PO Box 06440
Wacker Dr PO Sears Tower
Chicago, IL 60606
and
DRW Investments LLC
311 S Wacker Dr Ste 900
Chicago, IL 60606
ingber@ingber.com, ingber@alumni.caltech.edu
http://www.ingber.com/

ABSTRACT: A statistical mechanics of neuronal interactions (SMNI) is explored as providing some a basis for correlating EEG evoked potentials to reaction times. Some specific elements of SMNI, previously used to develop a theory of short-term memory (STM) and a model of electroencephalography (EEG) are key to providing this basis. Hick's Law, an observed linear relationship between reaction time (RT) and the information storage of STM is derived.

KEYWORDS: EEG; short term memory; nonlinear; statistical

1. Introduction

From circa 1978 through the present, a series of papers on the statistical mechanics of neocortical interactions (SMNI) has been developed to model columns and regions of neocortex, spanning mm to cm of tissue. Most of these papers have dealt explicitly with calculating properties of short-term memory (STM) and scalp EEG in order to test the basic formulation of this approach. SMNI derives aggregate behavior of experimentally observed columns of neurons from statistical electrical-chemical properties of synaptic interactions. While not useful to yield insights at the single neuron level, SMNI has demonstrated its capability in describing large-scale properties of short-term memory and electroencephalographic (EEG) systematics (Ingber, 1982; Ingber, 1983; Ingber, 1984; Ingber, 1991; Ingber, 1994; Ingber, 1995a; Ingber & Nunez, 1995; Ingber, 1996a; Ingber, 1997).

2. SMNI Description of Short-Term Memory (STM)

Since the early 1980's, a series of papers on the statistical mechanics of neocortical interactions (SMNI) has been developed to model columns and regions of neocortex, spanning mm to cm of tissue. Most of these papers have dealt explicitly with calculating properties of short-term memory (STM) and scalp EEG in order to test the basic formulation of this approach (Ingber, 1981; Ingber, 1982; Ingber, 1983; Ingber, 1984; Ingber, 1985a; Ingber, 1985b; Ingber, 1986; Ingber & Nunez, 1990; Ingber, 1991; Ingber, 1992; Ingber, 1994; Ingber & Nunez, 1995; Ingber, 1995a; Ingber, 1995b; Ingber, 1996b; Ingber, 1997; Ingber, 1998). This model was the first physical application of a nonlinear multivariate calculus developed by other mathematical physicists in the late 1970's (Graham, 1977; Langouche *et al*, 1982).

2.1. Statistical Aggregation

SMNI studies have detailed a physics of short-term memory and of (short-fiber contribution to) EEG phenomena (Ingber, 1984), in terms of M^G firings, where G represents E or I, M^E represents contributions to columnar firing from excitatory neurons, and M^I represents contributions to columnar firing from inhibitory neurons. About 100 neurons comprise a minicolumn (twice that number in visual cortex); about 1000 minicolumns comprise a macrocolumn. A mesocolumn is developed by SMNI to reflect the convergence of short-ranged (as well as long-ranged) interactions of macrocolumnar input on minicolumnar structures, in terms of synaptic interactions taking place among neurons (about 10,000 synapses per neuron). The SMNI papers give more details on this derivation.

In this SMNI development, a Lagrangian is explicitly defined from a derived probability distribution of mesocolumnar firings in terms of the M^G and electric potential variables, Φ^G. Several examples have been given to illustrate how the SMNI approach is complementary to other models. For example, a mechanical string model was first discussed as a simple analog of neocortical dynamics to illustrate the general ideas of top down and bottom up interactions (Nunez, 1989; Nunez & Srinivasan, 1993). SMNI was applied is to this simple mechanical system, illustrating how macroscopic variables are derived from small-scale variables (Ingber & Nunez, 1990).

2.2. STM Stability and Duration

SMNI has presented a model of STM, to the extent it offers stochastic bounds for this phenomena during focused selective attention. This 7 ± 2 “rule” is well verified by SMNI for acoustical STM (Ingber, 1984; Ingber, 1985b; Ingber, 1994), transpiring on the order of tenths of a second to seconds, limited to the retention of 7 ± 2 items (Miller, 1956). The 4 ± 2 “rule” also is well verified by SMNI for visual or semantic STM, which typically require longer times for rehearsal in an hypothesized articulatory loop of individual items, with a capacity that appears to be limited to 4 ± 2 (Zhang & Simon, 1985). SMNI has detailed these constraints in models of auditory and visual cortex (Ingber, 1984; Ingber, 1985b; Ingber, 1994; Ingber & Nunez, 1995).

Another interesting phenomenon of STM capacity explained by SMNI is the primacy versus recency effect in STM serial processing (Ingber, 1985b), wherein first-learned items are recalled most error-free, with last-learned items still more error-free than those in the middle (Murdock, 1983). The basic assumption is that a pattern of neuronal firing that persists for many τ cycles, τ on the order of 10 msec, is a candidate to store the “memory” of activity that gave rise to this pattern. If several firing patterns can simultaneously exist, then there is the capability of storing several memories. The short-time probability distribution derived for the neocortex is the primary tool to seek such firing patterns. The highest peaks of this probability distribution are more likely accessed than the others. They are more readily accessed and sustain their patterns against fluctuations more accurately than the others. The more recent memories or newer patterns may be presumed to be those having synaptic parameters more recently tuned and/or more actively rehearsed.

It has been noted that experimental data on velocities of propagation of long-ranged fibers (Nunez, 1981; Nunez, 1995) and derived velocities of propagation of information across local minicolumnar interactions (Ingber, 1982) yield comparable times scales of interactions across minicolumns of tenths of a second. Therefore, such phenomena as STM likely are inextricably dependent on interactions at local and global scales.

2.3. SMNI Correlates of STM and EEG

Previous SMNI studies have detailed that maximal numbers of attractors lie within the physical firing space of M^G, consistent with experimentally observed capacities of auditory and visual short-term memory (STM), when a "centering" mechanism is enforced by shifting background noise in synaptic interactions, consistent with experimental observations under conditions of selective attention (Mountcastle *et al*, 1981; Ingber, 1984; Ingber, 1985b; Ingber, 1994; Ingber & Nunez, 1995). This leads to all attractors of the short-time distribution lying along a diagonal line in M^G space, effectively defining a narrow parabolic trough containing these most likely firing states. This essentially collapses the 2 dimensional M^G space down to a 1 dimensional space of most importance. Thus, the predominant physics of short-term memory and of (short-fiber contribution to) EEG phenomena takes place in a narrow "parabolic trough" in M^G space, roughly along a diagonal line (Ingber, 1984).

Using the power of this formal structure, sets of EEG and evoked potential data, collected to investigate genetic predispositions to alcoholism, were fitted to an SMNI model to extract brain "signatures" of short-term memory (Ingber, 1997; Ingber, 1998). These results give strong quantitative support for an accurate intuitive picture, portraying neocortical interactions as having common algebraic or physics mechanisms that scale across quite disparate spatial scales and functional or behavioral phenomena, i.e., describing interactions among neurons, columns of neurons, and regional masses of neurons.

For future work, I have described how bottom-up neocortical models can be developed into eigenfunction expansions of probability distributions appropriate to describe short-term memory in the context of scalp EEG (Ingber, 2000). The mathematics of eigenfunctions are similar to the top-down eigenfunctions developed by some EEG analysts, albeit they have different physical manifestations. The bottom-up eigenfunctions are at the local mesocolumnar scale, whereas the top-down eigenfunctions are at the global regional scale. However, these approaches have regions of substantial overlap (Ingber & Nunez, 1990; Ingber, 1995a), and future studies may expand top-down eigenfunctions into the bottom-up eigenfunctions, yielding a model of scalp EEG that is ultimately expressed in terms of columnar states of neocortical processing of attention and short-term memory.

An optimistic outcome of future work might be that these EEG eigenfunctions, baselined to specific STM processes of individuals, could be a direct correlate to estimates of RT.

3. Hick's Law — Linearity of RT vs STM Information

The SMNI approach to STM gives a reasonable foundation to discuss RT and items in STM storage. These previous calculations support the intuitive description of items in STM storage as peaks in the 10-millisecond short-time (Ingber, 1984; Ingber, 1985b) as well as the several-second long-time (Ingber, 1994; Ingber & Nunez, 1995) conditional probability distribution of correlated firings of columns of neurons. These columnar firing states of STM tasks also were correlated to EEG observations of evoked potential activities (Ingber, 1997; Ingber, 1998). This distribution is explicitly calculated by respecting the nonlinear synaptic interactions among all possible combinatoric aggregates of columnar firing states (Ingber, 1982; Ingber, 1983).

The RT necessary to "visit" the states under control during the span of STM can be calculated as the mean time of "first passage" between multiple states of this distribution, in terms of the probability P as an outer integral $\int dt$ (sum) over refraction times of synaptic interactions during STM time t, and an inner integral $\int dM$ (sum) taken over the mesocolumnar firing states M (Risken, 1989), which has been explicitly calculated to be within observed STM time scales (Ingber, 1984),

$$RT = - \int dt \; t \int dM \; \frac{dP}{dt} \; . \tag{1}$$

As demonstrated by previous SMNI STM calculations, within tenths of a second, the conditional probability of visiting one state from another P, can be well approximated by a short-time probability distribution expressed in terms of the previously mentioned Lagrangian L as

$$P = \frac{1}{\sqrt{(2\pi dtg)}} \exp(-Ldt) \; , \tag{2}$$

where g is the determinant of the covariance matrix of the distribution P in the space of columnar firings.

This expression for RT can be approximately rewritten as

$$RT \approx K \int dt \int dM \; P \ln P \; , \tag{3}$$

where K is a constant when the Lagrangian is approximately constant over the time scales observed. Since the peaks of the most likely M states of P are to a very good approximation well-separated Gaussian peaks (Ingber, 1984), these states by be treated as independent entities under the integral. This last expression is essentially the "information" content weighted by the time during which processing of information is observed.

The calculation of the heights of peaks corresponding to most likely states includes the combinatoric factors of their possible columnar manifestations as well as the dynamics of synaptic and columnar interactions. In the approximation that we only consider the combinatorics of items of STM as contributing to most likely states measured by P, i.e., that P measures the frequency of occurrences of all possible combinations of these items, we obtain Hick's Law, the observed linear relationship of RT versus STM information storage. For example, when the bits of information are measured by the probability P being the frequency of accessing a given number of items in STM, the bits of information in 2, 4 and 8 states are given as approximately multiples of $\ln 2$ of items, i.e., $\ln 2$, $2 \ln 2$ and $3 \ln 2$, resp. (The limit of taking the logarithm of all combinations of independent items yields a constant times the sum over $p_i \ln p_i$, where p_i is the frequency of occurrence of item i.)

4. Conclusion

I have focussed on how bottom-up SMNI models can be developed into eigenfunction expansions of probability distributions appropriate to describe STM. This permits RT to be calculated as an expectation value over the STM probability distribution of stored states, and in the good approximation of such states being represented by well separated Gaussian peaks, this yield the observed linear relationship of RT versus STM information storage.

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