From Lagrangian To Laplacian: An Example From EEG Standing Waves

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ABSTRACT: This paper discusses the conversion of a Lagrangian to a Laplacian from a 2010 publication in Mathematical Biosciences (Ingber & Nunez, 2010). Another paper published in 1985 in IEEE Transactions in Biomedical Engineering performed similar calculations (Ingber, 1985a). In treating global mechanisms, we include myelinated axon propagation delays and periodic boundary conditions in the cortical-white matter system, topologically close to a spherical shell. The local mechanisms are multiscale interactions between cortical columns via short-ranged non-myelinated fibers. A mechanical model consisting of a stretched string with attached nonlinear springs demonstrates this idea. The string produces standing waves analogous to large-scale coherent EEG. The attached springs are analogous to the smaller mesoscopic columnar dynamics. A statistical mechanics of neocortical interactions (SMNI) calculates oscillatory behavior consistent with EEG, within columns, between neighboring columns via short-ranged non-myelinated fibers, and across cortical regions via myelinated fibers, to derive the string equation.

Keywords: EEG, nonlinear dynamics, standing waves, statistical mechanics, neocortical dynamics, short term memory

1. The Stretched String With Attached Springs

In order to distinguish theories of large-scale neocortical dynamics, we have proposed the label *local theory* to indicate mathematical models of cortical or thalamo-cortical interactions for which cortico-cortical axon propagation delays are assumed to be zero. The underlying time scales in these theories typically originate from membrane time constants giving rise to PSP rise and decay times. Thalamo-cortical networks are also “local” from the viewpoint of a surface electrode, which cannot distinguish purely cortical from thalamocortical networks. Finally, these theories are "local" in the sense of being independent of global boundary conditions dictated by the size and shape of the cortical-white matter system. By contrast, we adopt the label *global theory* to indicate mathematical models in which delays in the cortico-cortical fibers forming most of the white matter in humans provide the important underlying time scale for the large scale EEG dynamics recorded by scalp electrodes. Periodic boundary conditions are generally essential to global theories because the cortical-white matter system is topologically close to a spherical shell.

While this picture of distinct local and global models grossly oversimplifies expected genuine dynamic behaviors with substantial cross- scale interactions, it provides a convenient entry point to brain complexity. To facilitate our discussion, Figure 1 shows a stretched string with local stiffness (the little boxes) as a convenient dynamic metaphor (Nunez, 1995; Ingber, 1995a) The boxes might be simple linear springs with natural frequency ω_0 or they might represent nonlinear systems organized in a complex nested hierarchy. The proposed metaphorical relationships to neocortex are outlined in Table I and Figure 1. String displacement is governed by the basic string equation given by the Laplacian form

Basic String Equation

$$\frac{\partial^2 \Phi}{\partial t^2} - v^2 \frac{\partial^2 \Phi}{\partial x^2} + [\omega_0^2 + f(\Phi)]\Phi = 0 \tag{1}$$

For the simple case of homogeneous linear springs attached to a homogeneous linear string of length a and wave speed v, the normal modes of oscillation ω_n are given by

$$\omega_n^2 = \omega_0^2 + \left(\frac{n\pi v}{a}\right)^2 \qquad n = 1, 2, 3, \ldots \tag{2}$$

In this simple limiting case, the natural oscillation frequencies are seen as having distinct local and global contributions given by the first and second terms on the right side of the last equation, respectively. This same dispersion relation occurs for waves in hot plasmas and transmission lines, which might form closed loops more similar to the periodic boundary condition appropriate for neocortical standing waves. If the springs are disconnected, only the global dynamics remains. Or, if the string tension is relaxed, only the local dynamics remains. Next we approach the behavior of the nonlinear system described by the basic string equation, in which local and global effects are integrated.

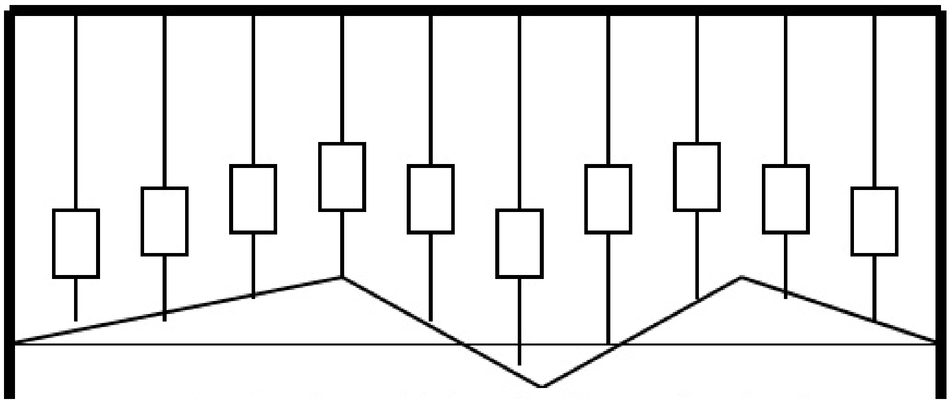

Fig. 1. The string-springs analog system. The small boxes might be simple linear springs or complex structures in a nested hierarchy analogous to columnar scale brain morphology (Ingber & Nunez, 2010).

String/Spring	**Neocortex/White Matter**
String displacement $\Phi(x, t)$	Any cortical field (synaptic, firing density)
String wave speed v	Cortico-cortico axon speed
Spring natural frequency ω_0	Simple cortico-thalamic feedback
Nonlinear stiffness $\omega_0^2 + f[\Phi(x, t)]$	Multiple-scale nonlinear columnar effects
Relax string tension $v \rightarrow 0$	Ignore axon delays
Disconnect boxes (springs) $\omega_0,\ f(\Phi) \rightarrow 0$	Ignore local dynamics

Table I. The string-springs system as a neocortical dynamic analog

2. Columnar Scales

Nature has developed structures at intermediate scales in many biological as well as in many non-biological systems to facilitate flows of information between relatively small and large scales of activity. Many systems possess such structures at so-called mesoscopic scales, intermediate between microscopic and macroscopic scales, where these scales are typically defined specific to each system, and where the mesoscopic scale typically facilitates information between the microscopic and macroscopic scales. Typically, these mesoscopic scales have their own interesting dynamics.

A statistical mechanics of neocortical interactions (SMNI) for human neocortex has been developed, building from synaptic interactions to minicolumnar, macrocolumnar, and regional interactions in neocortex (Ingber, 1982; Ingber, 1983). Over a span of about 40 years, a series of about 40 SMNI papers has been developed to model columns and regions of neocortex, spanning mm to cm of tissue. SMNI uses tools of nonlinear nonequilibrium multivariate statistical mechanics, a subfield of statistical mechanics dealing with Gaussian Markovian systems with time-dependent drifts and correlated diffusions, with both drifts and diffusions nonlinear in their multiple variables.

SMNI has described columnar activity to be an effective mesoscopic scale intermediate between macroscopic regional interactions and microscopic averaged synaptic and neuronal interactions. Such treatment of neuronal activity, beyond pools of individual neurons, is based on evidence of mesoscopic neocortical columnar anatomy as well as physiology which possess their own dynamics (Mountcastle, 1978; Buxhoeveden & Casanova, 2002). It is important to note that although columnar structure is ubiquitous in neocortex, it is by no means uniform nor is it so simple to define across many areas of the brain (Rakic, 2008). While SMNI has calculated phenomena like short-term memory (STM) and EEG to validate this model, there is as yet no specific real columnar data to validate SMNI's precise functional form at this scale. As found in the nature of intermediate scales in many chemical and biological systems, neuronal columnar structures display their own influences in neocortical information processing. For example, it has been proposed that interactions between minicolumns and complex glial networks, involve reciprocal magnetic interaction between neurons and astrocytes, influencing cerebral memory and computation (Banaclocha, 2007; Ingber, 2009b). There is ongoing research into algorithms that minicolumnar and macrocolumnar structures might use for neocortical information processing (Rinkus, 2010). This has included quantum effects (Ingber, 2018; Ingber, 2021). An example has been given of the use of nano-robots to deliver drugs targeted to specific molecular sites to aid STM.

When dealing with stochastic systems, there are several useful tools available when these systems can be described by Gaussian-Markovian probability distributions, even when they are in non-equilibrium, multivariate, and quite nonlinear in their means and variances. SMNI has demonstrated how most likely states described by such distributions can be calculated from the variational principle associated with systems, i.e., as Euler-Lagrange (EL) equations directly from the SMNI Lagrangian (Noether, 1918; Langouche *et al*, 1982). This Lagrangian is the argument in the exponent of the SMNI probability distribution. The EL equations are developed from a variational principle applied to this distribution, and they give rise to a nonlinear string model used by many neuroscientists to describe global oscillatory activity (Ingber, 1995a).

It is obvious that the mammalian brain is complex and processes information at many scales, and it has many interactions with sub-cortical structures. SMNI is appropriate to just a few scales and deals primarily with cortical structures. While SMNI has included some specific regional circuitry to address EEG calculations discussed below, details of laminar structure within minicolumns have not been included. Such laminar circuitry is of course important to many processes and, as stated in previous SMNI papers, it can be included by adding more variables. Some laminar structure is implicitly assumed in phenomena dealing with electromagnetic phenomena that depend on some systematic alignment of pyramidal neurons. Care has been taken to test SMNI at the appropriate scales, by calculating experimentally observed phenomena, and to some readers it may be surprising that it is so reasonably successful in these limited endeavors. The mathematics used is from a specialized area of multivariate nonlinear nonlinear nonequilibrium statistical mechanics (Langouche *et al*, 1982), and SMNI was the first physical application of these methods to the brain. In this paper, the mathematics used in all SMNI publications is not repeated, albeit referenced, but only enough mathematics is used to deal with the topic being presented. These EL equations are direct calculations of the nonlinear multivariate EL equations of the SMNI Lagrangian, giving most likely states of the system. The EL equations are quite general and are well known in physics for representing strings as well as springs, in simple as well as in complex stochastic systems, at both classical and quantum scales. This is the focus of this paper, to show how EEG may be conceptually viewed as a "string of springs."

3. SMNI

Neocortex has evolved to use minicolumns of neurons interacting via short-ranged interactions in macrocolumns, and interacting via long-ranged interactions across regions of macrocolumns. This common architecture processes patterns of information within and among different regions, e.g., sensory, motor, associative cortex, etc.

As depicted in Figure 2, SMNI develops three biophysical scales of neocortical interactions: (a)-(a*)-(a') microscopic neurons (Sommerhoff, 1974); (b)-(b') mesocolumnar domains (Mountcastle, 1978); (c)-(c') macroscopic regions. SMNI has developed conditional probability distributions at each level, aggregating up several levels of interactions. In (a*) synaptic inter-neuronal interactions, averaged over by mesocolumns, are phenomenologically described by the mean and variance of a distribution Ψ (both Poisson and Gaussian distributions were considered, giving similar results). Similarly, in (a) intraneuronal transmissions are phenomenologically described by the mean and variance of Γ (a Gaussian distribution). Mesocolumnar averaged excitatory (E) and inhibitory (I) neuronal firings M are represented in (a'). In (b) the vertical organization of minicolumns is sketched together with their horizontal stratification, yielding a physiological entity, the mesocolumn. In (b') the overlap of interacting mesocolumns at locations r and r' from times t and $t + \tau$ is sketched. Here τ~10 msec represents typical periods of columnar firings. This reflects on typical individual neuronal refractory periods of ~1 msec, during which another action potential cannot be initiated, and a relative refractory period of ~ 0.5—10 msec. Future research should determine which of these neuronal time scales are most dominant at the columnar time scale taken to be τ. In (c) macroscopic regions of neocortex are depicted as arising from many mesocolumnar domains. (c') sketches how regions may be coupled by long−ranged interactions.

Most of these papers have dealt explicitly with calculating properties of STM and scalp EEG in order to test the basic formulation of this approach (Ingber, 1982; Ingber, 1983; Ingber, 1984; Ingber, 1985a; Ingber, 1985b; Ingber & Nunez, 1990; Ingber, 1991; Ingber, 1994; Ingber & Nunez, 1995; Ingber, 1995a; Ingber, 1995b; Ingber, 1996; Ingber, 1997; Ingber, 1998; Ingber, 2018). The SMNI modeling of local mesocolumnar interactions, i.e., calculated to include convergence and divergence between minicolumnar and macrocolumnar interactions, was tested on STM phenomena. The SMNI modeling of macrocolumnar interactions across regions was tested on EEG phenomena.

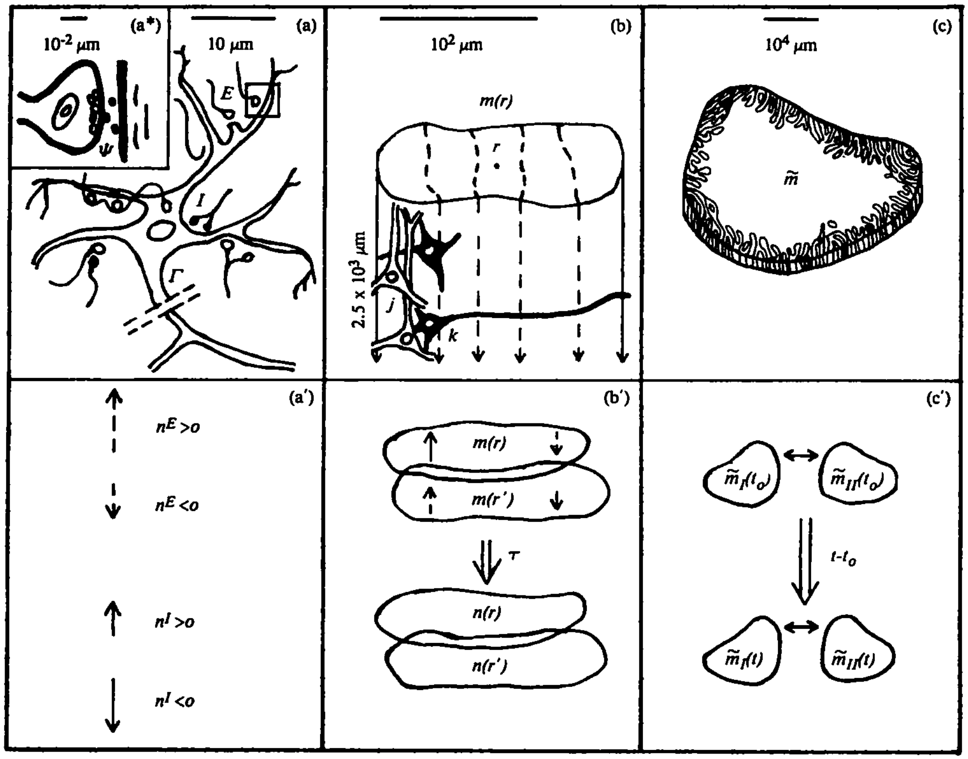

Fig. 2. Illustrated are three biophysical scales of neocortical interactions: (a)-(a*)-(a') microscopic neurons; (b)-(b') mesocolumnar domains; (c)-(c') macroscopic regions. Reprinted with permission from (Ingber, 1983) by the American Physical Society.

The EEG studies in previous SMNI applications were focused on regional scales of interactions. The STM applications were focused on columnar scales of interactions. However, this EEG study is focused at columnar scales, and it is relevant to stress the successes of this SMNI at this columnar scale, giving additional support to this SMNI model in this context. A previous report considered oscillations in quasi-linearized EL equations (Ingber, 2009a), while more recent studies consider the full nonlinear system (Ingber, 2009b).

4. Euler-Lagrange (EL)

The EL equations are derived from the long-time conditional probability distribution of columnar firings over all cortex, represented by $\tilde{M}$, in terms of the Action S, The path integral has a variational principle, $\delta L = 0$ which gives the EL equations for SMNI (Ingber, 1982; Ingber, 1983).

When dealing when multivariate Gaussian stochastic systems with nonlinear drifts and diffusions, it is possible to work with three essentially mathematically equivalent representations of the same physics: Langevin equations — coupled stochastic differential equations, a Fokker-Plank equation — a multivariate partial differential equation, and a path-integral Lagrangian — detailing the evolution of the short-time conditional probability distribution of the variables (Langouche *et al*, 1982).

While it typically takes more numerical and algebraic expertise to deal with the path-integral Lagrangian, there are many benefits, including intuitive numerical and algebraic tools. The Lagrangian components and EL equations are essentially the counterpart to classical dynamics,

Lagrangian Components Giving EL Equations

$$\text{Mass} = g_{GG'} = \frac{\partial^2 L}{\partial(\partial M^G/\partial t)\partial(\partial M^{G'}/\partial t)} \ ,$$

$$\text{Momentum} = \Pi^G = \frac{\partial L}{\partial(\partial M^G/\partial t)} \ ,$$

$$\text{Force} = \frac{\partial L}{\partial M^G} \ ,$$

$$\text{F} - \text{ma} = 0\text{:} \quad \delta L = 0 = \frac{\partial L}{\partial M^G} - \frac{\partial}{\partial t} \frac{\partial L}{\partial(\partial M^G/\partial t)} \tag{3}$$

The most-probable firing states derived from the variational principle from the path-integral Lagrangian as the EL equations represent a reasonable average over the noise in the SMNI system. For many studies, the noise cannot be simply disregarded, as demonstrated in other SMNI STM and EEG studies, but for the purpose here of demonstrating the existence of multiple local oscillatory states that can be identified with EEG frequencies, the EL equations serve very well.

The Lagrangian and associated EL equations have been developed at SMNI columnar scales, as well as for regional scalp EEG activity by scaling up from the SMNI columnar scales as outlined below.

4.1. Strings

The nonlinear string model was derived using the EL equation for the electric potential Φ measured by EEG, considering one firing variable along the parabolic trough of attractor states being proportional to Φ (Ingber & Nunez, 1990).

Since only one variable, the electric potential is being measured, is reasonable to assume that a single independent firing variable might offer a crude description of this physics. Furthermore, the scalp potential Φ can be considered to be a function of this firing variable. (Here, “potential” refers to the electric potential, not any potential term in the SMNI Lagrangian.)

Probabilities

In an abbreviated notation sub-scripting the time-dependence,

$$\Phi_t - \ll \Phi \gg = \Phi(M_t^E, M_t^I) \approx a(M_t^E - \ll M^E \gg) + b(M_t^I - \ll M^I \gg) \ , \quad (4)$$

where a and b are constants, and $\ll \Phi \gg$ and $\ll M^G \gg$ represent typical minima in the trough. In the context of fitting data to the dynamic variables, there are three effective constants, $\{ a, b, \phi \}$,

$$\Phi_t - \phi = aM_t^E + bM_t^I \quad (5)$$

Scaling

We scale and aggregate the mesoscopic columnar probability distributions, P, over this columnar firing space to obtain the macroscopic conditional probability distribution over the scalp-potential space:

$$P_\Phi[\Phi] = \int dM^E dM^I P[M^E, M^I] \delta[\Phi - \Phi'(M^E, M^I)] \tag{6}$$

Euler-Lagrange Equations

The EL equation includes variation across the spatial extent, x, of columns in regions,

$$\frac{\partial}{\partial t} \frac{\partial L}{\partial(\partial\Phi/\partial t)} + \frac{\partial}{\partial x} \frac{\partial L}{\partial(\partial\Phi/\partial x)} - \frac{\partial L}{\partial\Phi} = 0 \tag{7}$$

The result is

$$\alpha \frac{\partial^2\Phi}{\partial t^2} + \beta \frac{\partial^2\Phi}{\partial x^2} + \gamma\Phi - \frac{\partial F}{\partial\Phi} = 0 \tag{8}$$

The determinant prefactor g defined above also contains nonlinear details affecting the state of the system. Since g is often a small number, distortion of the scale of L is avoided by normalizing g/g_0, where g_0 is simply g evaluated at $M^E = M^{\ddagger E'} = M^I = 0$.

Recovering Basic String Equation

If there exist regions in neocortical parameter space such that we can identify $\beta / \alpha = -c^2$, $\gamma / \alpha = \omega_0^2$, i.e., as explicitly calculated using the CM and as derived in previous SMNI EEG papers,

$$\frac{1}{\alpha} \frac{\partial F}{\partial \Phi} = -\Phi f(\Phi) , \tag{9}$$

then we recover the nonlinear string model.

4.2. Springs

For a given column in terms of the probability description given above, the above EL equations are represented as

$$\frac{\partial}{\partial t} \frac{\partial L}{\partial(\partial M^E / \partial t)} - \frac{\partial L}{\partial M^E} = 0 \ ,$$

$$\frac{\partial}{\partial t} \frac{\partial L}{\partial(\partial M^I / \partial t)} - \frac{\partial L}{\partial M^I} = 0 \qquad (10)$$

To investigate dynamics of multivariate stochastic nonlinear systems, such as neocortex presents, it is not sensible to simply apply simple mean-field theories which assume sharply peaked distributions, since the dynamics of nonlinear diffusions in particular are typically washed out.

Previous SMNI EEG studies had demonstrated that simple linearized dispersion relations derived from the EL equations support the local generation of frequencies observed experimentally as well as deriving diffusive propagation velocities of information across minicolumns consistent with other experimental studies. The earliest studies simply used a driving force $J_G M^G$ in the Lagrangian to model long-ranged interactions among fibers (Ingber, 1982; Ingber, 1983). Subsequent studies considered regional interactions driving localized columnar activity within these regions (Ingber, 1996; Ingber, 1997; Ingber, 1998).

A recent set of calculations examined these columnar EL equations to see if EEG oscillatory behavior could be supported at just this columnar scale, i.e., within a single column. At first, the EL equations were quasi-linearized, by extracting coefficients of M and dM/dt. The nonlinear coefficients were presented as graphs over all firing states (Ingber, 2009a). This exercise demonstrated that a spring-type model of oscillations was plausible. Then a more detailed study was performed, developing over two million lines of C code from the algebra generated by an algebraic tool, Maxima, to see what range of oscillatory behavior could be considered as optimal solutions satisfying the EL equations (Ingber, 2009b). The answer was affirmative, in that ranges of $\omega t \approx 1$ were supported, implying that oscillatory solutions might be sustainable just due to columnar dynamics at that scale. Below, the full probability distribution is evolved with such oscillatory states, confirming this is true.

To understand the nature of the EL equations, it is useful to view the probability space over which most likely states exist. Figure 3 presents the probability distribution over firing space, as the PATHINT code evolves it over 1 sec, folding every $\tau/10$, or 1000 folding to reach 1 sec. Similar to the previous calculations (Ingber & Nunez, 1995), to control large negative drifts at the boundaries which can cause anomalous numerical problems at the edges of firing space, a simple Gaussian cutoff with width 0.1 was taken for the drifts at those boundaries. This cutoff was applied as well to the diffusions, which kept the mesh uniform near the edges, resulting in the over all mesh being within 1-3 (2-4) firing units for non-visual (visual) Cases. This coarse mesh results in some fine structure in the trough not visible in the graphs, but the overall structure of the distributions are clear. Each folding took about 0.045 (0.16) secs on a dedicated IBM a31p Thinkpad with 1 GB RAM running at 1.8 GHz for non-visual (visual) Cases, under gcc/g++-4.3.3 under Linux Ubuntu 9.04. The kernel is a banded matrix with over 250K (500K) non-zero entries for non-visual (visual) Cases. It is clear that columnar STM under all 4 Cases is quite stable for at least 1 sec. Similar results are obtained for Cases BCV, EC and IC.

5. Numerical Details

In 1985 the author published a paper (Ingber, 1985a) on "EEG Dispersion Relations" using the linearized Euler-Lagrange equations to derive a dispersion relation giving numerical details of the frequencies calculated by SMNI, yielding frequencies close to the "alpha" rhythm.

At time this author was not aware that the EEG community had or had already a "wave equation" used to describe EEG. Nevertheless, the tools described here were used as described.

As stated in that Abstract

> An approach is explicitly formulated to blend a local with a global theory to investigate oscillatory neocortical firings, to determine the source and the information-processing nature of the alpha rhythm. The basis of this optimism is founded on a statistical mechanical theory of neocortical interactions which has had success in numerically detailing properties of short-term-memory (STM) capacity at the mesoscopic scales of columnar interactions, and which is consistent with other theory deriving similar dispersion relations at the macroscopic scales of electroencephalographic (EEG) and magnetoencephalographic (MEG) activity.

The details are given in an Appendix A:

Euler-Lagrange Variational Equations. The Euler-Lagrange variational equations associated with $\underline{L}_F$ leads to a set of 12 coupled first-order differential equations, with coefficients nonlinear in M^G , in the 12 variables $\{M^G, \dot{M}^G, \ddot{M}^G, \nabla M^G, \nabla^2 M^G\}$ in $(r;t)$ space. In the neighborhood of extrema $\ll \bar{M}^G \gg$, $\underline{L}_F$ can be expanded as a Ginzburg-Landau polynomial. To investigate first-order linear oscillatory states, only powers up to 2 in each variable are kept, and from this the variational principle leads to a relatively simple set of coupled linear differential equations with constant coefficients:

$$0 = \hat{\delta} \underline{L}_F = \underline{L}_{F,\dot{G}:t} - \hat{\delta}_G \underline{L}_F \tag{A6}$$

$$\approx - \underline{f}_{|G|} \ddot{\underline{M}}^{|G|} + \underline{f}^1_G \dot{\underline{M}}^{G\neg} - \underline{g}_{|G|} \nabla^2 \underline{M}^{|G|} + \underline{b}_{|G|} \underline{M}^{|G|} + \underline{b}\, \underline{M}^{G\neg} \ , \ G\neg \ne G \ ,$$

$$[\cdots]_{,\dot{G}:t} = [\cdots]_{,\dot{G}G'} \dot{M}^{G'} + [\cdots]_{,\dot{G}\dot{G}'} \ddot{M}^{G'} \ ,$$

$$\underline{M}^G = M^G - \ll \bar{M}^G \gg \ , \ \underline{f}^1_E = -\underline{f}^1_I \equiv \underline{f} \ .$$

These equations are then Fourier transformed and the resulting dispersion relation is examined to determine for which values of the synaptic parameters and of $\underline{\xi}$, the conjugate variable to r, can oscillatory states, $\omega(\underline{\xi})$, persist. E.g., solutions are sought of the form

$$\underline{M}^G = \mathrm{Re}\, \underline{M}^G_{\mathrm{osc}} \exp[-i(\underline{\xi} \cdot r - \omega t)] \ , \tag{A7}$$

$$\underline{M}^G_{\mathrm{osc}}(r,t) = \int d^2\underline{\xi} d\omega\, \hat{\underline{M}}^G_{\mathrm{osc}}(\underline{\xi},\omega) \exp[i(\underline{\xi} \cdot r - \omega t)] \ .$$

For instance, a typical example is specified by: extrinsic sources $J_E = -2.63$ and $J_I = 4.94$, $N^E = 125$, $N^I = 25$, $V^G = 10$ mV, $A^E = 1.75$, $A^I = 1.25$, $B^G = 0.25$, and $v^G = \phi^G = 0.1$ mV. The global minima is at $\bar{M}^E = 25$ and $\bar{M}^I = 5$. This set of conditions yields (dispersive) dispersion relations

$$\omega\tau = \pm\{ -1.86 + 2.38(\xi\rho)^2; -1.25i + 1.51i(\xi\rho)^2 \} \ , \tag{A8}$$

where $\xi = |\underline{\xi}|$. The propagation velocity defined by $d\omega/d\xi$ is ~1 cm/sec, taking typical wave-numbers ξ to correspond to macrocolumnar distances ~30ρ. Calculated frequencies ω are on the order of EEG frequencies ~10^2 sec^{-1}. These mesoscopic propagation velocities permit processing over several minicolumns ~10^{-1} cm, simultaneous with processing of mesoscopic interactions over tens of cm via association fibers with propagation velocities ~600—900 cm/sec. I.e., both can occur within ~10^{-1} sec.

6. Summary and Conclusion

We have suggested that dynamic behavior in neocortex is due to some combination of global and local processes with important top-down and bottom-up interactions across spatial scales, a typical feature of many if not most complex physical, biological, social, financial, and other systems. We have focused on electroencephalography (EEG) because EEG provides most of the existing data on the relationship between ms scale neocortical dynamics and brain state. Although EEG recorded from the human scalp provides data at very large spatial scales (several cm), it is closely correlated with many distinct kinds of cognitive processing.

A purely global EEG model stresses myelinated axon propagation delays and periodic boundary conditions in the cortical-white matter system. As this system is topologically close to a spherical shell, standing waves are predicted with fundamental frequency in the typical EEG range near 10 Hz. In sharp contrast to the purely global model, the proposed local mechanisms are multiscale interactions between cortical columns via short-ranged non-myelinated fibers. A statistical mechanics of neocortical interactions (SMNI) predicts oscillatory behavior within columns, between neighboring columns and via short-ranged non-myelinated fibers. The columnar dynamics, based partly on membrane time constants, also predicts frequencies in the range of EEG.

We generally expect both local and global processes to influence EEG at all scales, including the large scale scalp data. Thus, SMNI also includes interactions across cortical regions via myelinated fibers effecting coupling the local and global models. The combined local-global dynamics is demonstrated with an analog mechanical system consisting of a stretch string (producing standing waves) with attached nonlinear springs representing columnar dynamics. SMNI is able to derive a string equation consistent with the global EEG model. We conclude that the string-spring system provides an excellent analog with several general features that parallel multiscale interactions in genuine neocortex.

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