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STATISTICAL MECHANICS OF NEOCORTICAL INTERACTIONS.
I. BASIC FORMULATION

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An approach to collective aspects of the neocortical system is formulated by methods of modern nonlinear nonequilibrium statistical mechanics. Microscopic neuronal synaptic interactions, consistent with anatomical observations, are first spatially averaged over columnar domains. These spatially ordered domains retain contact with the original physical synaptic parameters, are consistent with observed columnar physiology, and are a suitable substrate for macroscopic spatial-temporal regions described by a Lagrangian formalism. Long-ranged influences from extrinsic and inter-regional afferents drive these short-ranged interactions, giving rise to several columnar mechanisms affecting macroscopic activity.

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1. Introduction

A theory is developed to describe neocortical activity occurring in large regions of neocortex, involving millions to billions of neurons, transpiring for hundreds of msec. Input is a set of chemical and electrical parameters derived from properties of microscopic synaptic interactions formulated in Sec. 2. Analytic derivations and precise definition of these columnar domains is formulated in Sec. 3. A stochastic treatment of the firings of these ordered columns permits the macroscopic development of intrinsic regional activity formulated in Sec. 4. Numerical work that concretizes this formalism is given in Sec. 5.

A major contribution of this work is the analytic treatment of minicolumns [81]. Minicolumns are observed to span $\approx 7 \times 10^2 \mu\text{m}^2$. Mesocolumnar domains are defined here as the spatial extent of minicolumns, in order to distinguish their scale from that of microscopic neurons. The calculations in Secs. 2 and 5 support observations of periodically alternating firings of columnar structures [29,32,46,47,53]. As pictured in Fig. 1, this microscopic scale is orders of magnitude larger than the molecular scale which enters membrane physics. Also note that "macrocolumns" spanning $\approx 7 \times 10^5 \mu\text{m}^2$ have been defined as another physiological entity observed in neocortex [81], but the macroscopic regions considered here are orders of magnitude larger than these. Mesocolumnar domains are sufficiently close to the scale of microscopic neurons to allow direct dependence of this theory on neuronal chemical and electrical properties. The proper stochastic treatment of their interaction permits their development into macroscopic regions responsible for global neocortical information processing. "Thermodynamic" entities corresponding to the "free energy" potential, "temperature," and order parameters of these macroscopic regions are derived by a statistical mechanics paradigm [62].

1.1. *Rationale*

Relative to other biological entities, the intrinsic synaptic activity of the most highly evolved mammalian human neocortex functions via the most degenerate and the shortest-ranged neuronal interactions ($\approx \mu$m). This suggests that many collective aspects of this system may be fruitfully studied similarly to other collective systems. Collective effects, from clustering [102,103] or from statistical interactions [55], are proposed to be mechanisms of information processing, in addition to the "hard-wiring" mechanisms also possessed by other more ordered cortical entities [3,100].

-- Fig. 1 --

Reasonable criteria for any physical approach to neocortex should include three basic Features. These also serve to illustrate the appropriate analogies between neocortex and other collective physical systems.

(A) Interactions. Short-ranged neuronal interactions over time periods of several msec should be derived from even more microscopic synaptic activities [98]. (See Fig. 1a.) Long-ranged spatial interactions from specific neuronal pathways, primarily composed of the relatively low population of long excitatory fibers from ipsilateral association, contralateral commissural, and thalamocortical processes must be consistently treated. These long-ranged interactions are also important for collective activity in mammalian cortex [13], and they are included in this study. Longer-time, weaker and modulatory nonsynaptic influences arising from humoral and electrotonic interactions [26,77,92] are also included, but only as their averaged properties affect synaptic parameters.

(B) Statistics. Neurons separated by large distances, across 10^3 to 10^8 neurons, can be statistically coupled via the short-ranged interactions in (A). (See Fig. 1c.) Order parameters must be identified, and intrinsic fluctuations from the microscopic synaptic and neuronal systems, diffusion effects, must be included. There are also fluctuations of the mesoscopic system due to their interactions, derived here as gradient couplings between neighboring mesoscopic cells. These spatially ordered mesoscopic domains respect the observed anatomy and physiology of neocortex [102,103] more than earlier theories hypothesising random neural networks [39,76].

(C) Dynamics. A viable formalism must be adopted to describe the statistical time evolution of the macroscopic nonequilibrium system over scales of 10^2 to 10^4 msec.

Although cooperativity between distant neurons is typically quite low [2], macroscopic regions reflect cooperative behavior, proposed here to best be understood as initiated at the mesoscopic level of interaction. The empirical existence of collective spatial-temporal activity, embedded in a spontaneous background orders of magnitude larger, is supported by statistical analyses of electroencephalographic and magnetoencephalographic recordings [52,112]. Evoked potential studies [43,61,82] confirm the existence of macroscopic activity persisting for hundreds of msec [27]. Some studies suggest that much of EEG data from electromagnetic potentials on the scalp can be explained as dynamic dipole-layered activity, driven by the relatively low population of long-ranged excitatory fibers, only somewhat mediated by the short-ranged interactions [86,87]. Although this hypothesis probably understates the the degree to which EEG is a measure of neocortical activity, this does imply that some EEG data may not be good physiological correlates of much of neocortical information processing which is certainly mediated by microscopic and mesoscopic interactions. Numerical calculations in Sec. 5 give insight into these observations.

Ultimately, coordinated studies using behavioral control, external stimulation and single neuron analyses, in many regions of neocortex, must provide better empirical data regarding collective neural mechanisms. For example, consistent with previous investigations, current studies [80] can not discern if specific neuronal connections exist to explain ambient (global) visual attention, which is parallel processed with focal attention. This present study suggests that collective columnar mechanisms should be investigated as candidates to explain such global phenomena.

As long as collective mechanisms arising in a physical system characterized by the above three Features are considered to be viable sources of collective neocortical phenomena, then these Features must be correctly formulated.

1.2. *Other Studies*

There is a large literature dealing with neuronal mechanisms that intuits phenomenological differential equations from rates of change of average excitatory and inhibitory neuronal firings, and then proceeds to search for collective behavior, limit cycles, and oscillatory behavior [8,30,54,113-116]. Mechanisms are sought to explain varied phenomena such as hysteresis in perception [114], perception and learning [42,104], and ontogenesis of columnar formation [104,109].

Comparisons with applications of these techniques to those used in other physical systems [40,41], illustrates that the pioneering application of these appropriate formalisms to the neocortical system still has much to offer. Much inspiration for these applications has come from work in nonequilibrium thermodynamics [40,41,55,60,84,106]. The statistical physics formulation of similar systems is, however, less phenomenological and more useful for this study. Important nonlinearities are also properly treated. This has not been true in previous studies, and they are therefore incapable of performing detailed analyses and calculations as performed in Secs. 2-5. The last two subsections of Sec. 4, although not essential for this development, rigorously compare this study to other studies.

-- Fig. 2 --

This study also distinguishes between neuronal mechanisms the neocortex uses to process information, from the structures of information the neocortex processes. (See Fig. 2.) An Onsager-Machlup type Lagrangian [88,89], or "potential," is derived which operates on firings of the system. The exponential of minus the Lagrangian is a weighting factor on all possible states, filtering or transforming (patterns of) input firings into output firings. "Information" is a concept well defined in Sec. 4, in terms of the probability eigenfunctions of electrical-chemical activity of this Lagrangian.

Another large literature dealing with information processing of neural networks, applied to perception, learning and memory, typically intuits the existence of an algebra of information-vectors isomorphic to a linear vector space [4,6,7,17]. The details of the structures of these vectors and of their relationships are based on the empirics of neuronal interactions and of hypothesised patterns of neuronal firings. The experimental and theoretical value of their conclusions are generally diminished because they do not reasonably draw empirical support and analytic development from relatively fundamental physical interactions, from Features (A), (B) or (C). In this study, the sets of eigenfunctions of the physical Lagrangian which includes these Features are candidates for the linear vector spaces previously assumed by these other studies.

2. Microscopic Neurons

Fig. 1a illustrates the microscopic neuronal interaction scale, on the order of several μm. Neocortical neurons typically have many dendrites that receive quanta of chemical postsynaptic stimulation from many other neurons. The distribution of quanta transmitted across synapses takes place on the scale of $10^{-2}\,\mu$m, as illustrated in the inset Fig. 1a*. Each quantum has thousands of molecules of chemical neurotransmitter that affect the chemically-gated postsynaptic membrane. Chemical transmissions in neocortex are believed to be either excitatory (E), such as glutamic acid, or inhibitory (I), such as γ-aminobutyric acid. There exist many transmitters as well as other chemicals that modulate their effects, but it is assumed that after millions of synapses between hundreds of neurons are averaged over, then it is reasonable to ascribe a distribution Ψ with a mean and variance for E and I inter-neuronal interactions.

This same averaging procedure makes it reasonable to ascribe a distribution Γ with a mean and variance for (E) and (I) intra-neuronal interactions. A Gaussian Γ is taken to describe the distribution of electrical polarizations caused by chemical quanta impinging on the postsynaptic membrane. These polarizations give a resultant polarization at the base of the neuron, the axon (extension in Fig. 1a cut by the double broken line). The base of the axon of a large fiber may be myelinated. However, smaller neurons typically lack these distinguishing features. Experimental techniques are not yet sufficiently advanced to attempt the explicit averaging procedure necessary to establish the means and variances of Ψ and Γ, and their parameters *in vivo.* Differential attenuations of polarizations from synapses to the base of an axon are here only phenomenologically accounted for by including these geometric and physiological effects into Γ.

With a sufficient depolarization of ≈ 10 to 20 mV, within an absolute and relative refractory period of ≈ 5 msec, an action potential is pulsed down the axon and its many collaterals, affecting voltage-gated presynaptic membranes to release quanta of neurotransmitter. Not detailed here is the biophysics of membranes, of thickness $\approx 5 \times 10^{-3}\,\mu$m, composed of biomolecular leaflets of phospholipid molecules [15,95,107,108]. At present, Ψ and Γ are taken to approximate this biophysics for use in macroscopic studies. The formalism adopted in this study is capable of using new microscopic functional dependences, gleaned from other empirical or theoretical investigations, and cranking them through to obtain similar macroscopic descriptions. Chemical independences of excitatory depolarizations and inhibitory hyperpolarizations are well established in neocortex, and this independence is retained throughout this study.

It should be noted that experimental studies used to infer Ψ and Γ (at neuromuscular junctions) were made possible by deliberately reducing the number of quanta by lowering external Ca concentrations [12,56]. Ψ was found to be Poisson, but in that system, where hundreds of quanta are transmitted *in vivo*, Ψ may well be otherwise, for example Gaussian with independent mean and variance. Current research suggests a binomial distribution, having a Poisson limit [50,57,90]. In neocortex, probably small numbers of quanta are transmitted at synapses, but other effects, such as nonuniformity and nonstationarity of presynaptic release sites, and nonlinear summation of postsynaptic potentials, may detract from a simple phenomenological Poisson description [98]. This short description serves to point out possible differences in Ψ from many sources. However, the derivation of synaptic interactions given here makes it plausible that for reasonable neuronal parameters, the statistical folding of Ψ and Γ is essentially independent of the functional form, not the numerical mean and variance, taken for Ψ.

The result of this analysis is to calculate the transition probability of the firing of neuron j, p_{σ_j}, given its interaction with its neighbors that also may fire or not fire. Eq. (2.12) gives the result as the tabulated error function. Within the range where the total influences of excitatory and inhibitory firings match and exceed the average threshold potential of a given neuron, the probability of that neuron firing receives its major contribution to increase from 0 towards 1. A step-function derived as $\tanh F^G$ in Eq. (3.13) is defined by the "threshold factor" F_j in Eq. (2.12). The mesoscopic development discussed below retains this sensitivity.

This is similar to the mathematical result obtained by others [67,68,96,97] who have modelled neocortex after magnetic systems [18,83]. However, the following is derived more generally, and has the neural parameters more specifically denoted with different statistical significances given to Ψ and Γ as described above. The severe approximations made in previous studies to obtain tractable algebraic expressions are not necessary in this study. Additionally, Features (B) and (C) are correctly developed here.

2.1. *Conditional Probability*

Consider $10^2 < N < 10^3$ neurons, labelled by k, interacting with a given neuron j. Each neuron may contribute many synaptic interactions to many other neurons. A neuron may have as many as 10^4 synaptic interactions. Within time $\tau_n \approx 5$ msec, Ψ is the distribution of q quanta of chemical transmitter released from neuron k to neuron $j (k \neq j)$ with mean a_{jk}, where

$$a_{jk} = A_{jk}(\sigma_k + 1)/2 + B_{jk} ; \tag{2.1}$$

A_{jk} is the conductivity weighting transmission of polarization, dependent on k firing,

$$\sigma_k = \begin{cases} 1, & k \text{ fires,} \\ -1, & k \text{ doesn't fire,} \end{cases} \tag{2.2}$$

and B_{jk} is a background including some nonsynaptic and long-range activity. Of course, A and B are highly complicated functions of ij. This definition of σ_k permits a decomposition of a_{jk} into two different physical contributions.

Within the scope of the assumption that postsynaptic potential responses from numbers of presynaptic released quanta add algebraically, a Gaussian process is taken to represent this response for each quantum released. Application of the central limit theorem [70] then yields, for any q quanta, a Gaussian process Γ for imparting a potential W_{jk} to neuron j:

$$\Gamma = (2\pi q \phi_{jk}^2)^{-\frac{1}{2}} \exp[-(W_{jk} - q v_{jk})^2]/(2q\phi_{jk}^2) , \tag{2.3}$$

$$\lim_{q \to 0} \Gamma \equiv \delta(W_{jk}) ,$$

where the polarization v_{jk} can be positive (excitatory E) or negative (inhibitory I), and

$$\delta(Z) = (2\pi)^{-1} \int_{-\infty}^{\infty} dQ \; exp(iQZ) \tag{2.4}$$

represents a well-behaved, strongly peaked distribution.

The probability S_{jk} of developing W_{jk} from k is

$$S_{jk} = \sum_{q=0}^{\infty} \Gamma\Psi . \tag{2.5}$$

The probability S_j of developing potential W_j from all N neurons is

$$S_j = \int \cdots \int dW_{j1} \ldots dW_{jN} \; S_{j1} \ldots S_{jN} \delta(W_j - \sum_k W_{jk}) . \tag{2.6}$$

The conditional probability p_{+j} of neuron j firing if $W_j > V_j$, the threshold of j, is

$$p_{+j} = \int_{V_j}^{\infty} dW_j S_j \ . \tag{2.7}$$

At this stage it is possible to include the probability of not firing by defining

$$p_{\sigma_j} = \begin{cases} p_{+j}, \ \sigma_j = +1 \\ p_{-j}, \ \sigma_j = -1, \end{cases} \tag{2.8a}$$

by replacing

$$V_j \to \sigma_j V_j, \ Q \to \sigma_j Q \tag{2.8b}$$

in Eqs. (2.7) and (2.4).

2.2. *Poisson* Ψ

For a_{jk} small, take Ψ as Poisson with mean and variance a_{jk}:

$$\Psi = exp(-a_{jk})(a_{jk})^q/q!. \tag{2.9}$$

Eq. (2.7) yields:

$$p_{\sigma_j} = \int_{V_j}^{\infty} dW_j (2\pi)^{-1} \int_{-\infty}^{\infty} dQ exp(iQW_j) \tag{2.10}$$

$$\times \exp\{-\sum_k a_{jk}[1 - exp(-i\sigma_j Q v_{jk} - Q^2 \phi_{jk}^2/2)]\}.$$

An extremum approximation is now valid: The main contribution to p_{σ_j} comes from non-oscillatory contributions from the second exp in Eq. (2.10), where its argument has a minimum, rendering it Gaussian. Using

$$\phi_{jk} < |v_{jk}| \ll V_j \le W_j \ , \tag{2.11}$$

p_{σ_j} may be calculated as

$$p_{\sigma_j} = \pi^{-\frac{1}{2}} \int_{(\sigma_j F_j \sqrt{\pi}/2)}^{\infty} dz \ exp(-z^2) \tag{2.12}$$

$$= \tfrac{1}{2}\,[1 - \mathrm{erf}(\sigma_j F_j \sqrt{\pi}/2)],$$

$$F_j = (V_j - \sum_k a_{jk} v_{jk})/[\pi \sum_{k'} a_{jk'}(v_{jk'}^2 + \phi_{jk'}^2)]^{\frac{1}{2}} \ . \tag{2.13}$$

"erf" is the tabulated error function, simply related to the "normal probability function" [51,70]. F_j is a "threshold factor," as p_{σ_j} increases from 0 to 1 between $\infty > \sigma_j F_j > -\infty$ sharply within the range of $F_j \approx 0$.

2.3. *Gaussian* Ψ

The mean of a Poisson distribution of q successes is

$$a_{jk} = \psi \bar{e} \ , \tag{2.14}$$

$\bar{e}$ the large number of repetitions of an "experiment," likely correlated with the number of synaptic knobs [50,57], and ψ the small probability of success, the average probability of release of one quantum. For large a_{jk}, a Gaussian Ψ' representing Ψ is defined with mean a_{jk} and variance $a_{jk}(1 - \psi)$ [70]:

$$\Psi' = [2a_{jk}(1-\psi)]^{-\frac{1}{2}} exp[-(q-a_{jk})^2/(2a_{jk}(1-\psi))] \tag{2.15}$$

$$/ \int_{-[a_{jk}/(2(1-\psi))]^{\frac{1}{2}}}^{\infty} dz exp(-z^2) .$$

Take

$$\sum_q \to \int_0^\infty dq \tag{2.16}$$

in Eq. (2.5). Again, using Eq. (2.11), derive Eq. (2.12), but with

$$v^2_{jk'} \to (1-\psi)v^2_{jk'} \ . \tag{2.17}$$

2.4. *Arbitrary* Ψ

Examination of this derivation shows that Eq. (2.12) results from the folding of Γ and Ψ, for a wide range of reasonable Ψ peaked near $q = a_{jk}$.

If

$$|\sigma_j F_j| < 1 , \tag{2.18}$$

to be validated in Sec. 3, then an asymptotic expression for Eq. (2.12) is

$$p_{\sigma_j} \approx exp(-\sigma_j F_j)/(exp\ F_j \ + \ exp - F_j) . \tag{2.19}$$

This form of p_{σ_j} exposes the linear dependence of the argument on σ_j and F_j. Apparently, it has not been appreciated just how good an approximation $\tanh(2z/\sqrt{\pi})$ is to erf (z). Using the notation (z, tanh / erf), precisely obtain: (0., 0. / 0.), (.01, .0113 / .0113), (.1, .1124 / .1125), (.5, .5111 / .5205), (1., .8105 / .8427), (2., .9783 / .9953), (2.5, .9929 / .9996), (∞, 1. / 1.). For small z obtain:

$$\tanh(2z/\sqrt{\pi}) = (2/\sqrt{\pi})(z - z^3/2.36 + z^5/4.63 - z^7/8.98 \pm \cdots) \tag{2.20}$$

$$erf(z) = (2/\sqrt{\pi})(z \ - z^3/3 \ + z^5/10 \ - z^7/42 \ \pm \cdots) .$$

3. Mesoscopic Domains

At this stage, severe approximation in modelling would have to be, and usually is attempted. However, advantage can be taken of empirically observed columnar structure to first attempt to analytically scale the neuronal system into mesoscopic domains that are still relatively microscopic to the macroscopic regions to be described [29,32,47,53,81,103,104]. For purposes of macroscopic description, the minicolumnar structure effectively spatially averages the neuronal interactions within one to several firing periods.

The following development: (1) reasonably includes and averages over millions of synaptic interactions that exist between groups of hundreds of neurons; (2) analytically establishes the integrity of columnar domains and specifies their interactions; (3) prepares the formulation of (1) and (2) to foresee their analytic inclusion into studies of macroscopic regions.

The neocortex has $\approx 5 \times 10^{10}$ neurons distributed rather uniformly over $\approx 5 \times 10^8$ minicolumns. (The visual cortex has double this density.) Within these minicolumns, a "vertical" structure is defined perpendicular to six highly convoluted laminae of total thickness $\approx 2.5 \times 10^3 \mu$m, principally by the efferent pyramidal cells. They exhibit vertical apical bundling of their dendrites in the upper laminae, and some of their recurrent axonal collaterals also ascend to upper laminae. A number of other fusiform, Martinotti, and stellate cells (granule cells in sensory cortex and basket cells in motor cortex) also contribute to this vertical organization. In general, laminae I to IV are afferent and laminae V and VI are efferent [3].

However, "horizontal" dendritic basal arborizations (tree-like structures) of the pyramidal cells, tangential to the laminae, horizontal axonal collaterals of the pyramidal cells, and horizontal processes of stellate, Martinotti, and neonatal horizontal cells, all impart horizontal stratification to columnar interactions. Therefore, although the columnar concept has anatomical and physiological support, the minicolumnar boundaries are not so clearly defined [27]. This stratification and other long-ranged afferent input to groups of minicolumns define a physiological unit that encompasses a mesocolumn, consisting of one to perhaps several minicolumns.

This study formalizes these circumstances by defining a mesocolumn with extent $> 10^2 \mu$m, as an intermediate integral physiological unit encompassing one to several minicolumns. (See Fig. 1b.) Dynamic nearest-neighbor interactions between mesocolumns are analytically defined by their overlapping neuronal interactions, in accordance with the observed horizontal columnar stratifications outlined above. (See Fig. 1b’.) This approach permits future analytic modifications, as differences between inter- and intra-minicolumnar interactions and circuitries become experimentally clarified.

The resulting picture of columnar interactions is relatively simpler than a mass of neurons, but not so simple to the point of uselessness. A collection of average excitatory and inhibitory neuronal firings, as depicted in Fig. 1a’, now define a continuum of mesocolumnar firings. A zero order binomial distribution is easily intuited: Let G denote E or I firings. A column of N^G neurons can have a total firing of Nn^G, where n^G is the fraction firing, ranging by 2’s between $-N^G \le Nn^G \le N^G$. (Count firing as +1, non-firing as -1.) For convenience, assume $Nn^G > 0$, which arises from Nn^G firings plus $\frac{1}{2}(N^G - Nn^G)$ cancelling pairs of firings and non-firings. This gives a total of $\frac{1}{2}(N^G - Nn^G) + Nn^G = \frac{1}{2}(N^G + Nn^G)$ firings and $N^G - \frac{1}{2}(N^G + Nn^G)$ non-firings. The degeneracy factor, as a function of the firing rate Nn^G, is the number of ways N^G neurons can produce a given firing pattern. The final answer is the binomial distribution, Eq. (3.12). Note that the binomial coefficient is unity for states of all firing or all non-firing, and peaks as $N^G!/[(N^G/2)!]^2 \approx 2^{N^G + \frac{1}{2}}(\pi N^G)^{-\frac{1}{2}}$ for $Nn^G = 0$. In the range $Nn^G \approx 0$, there is maximal degeneracy of information encoded by mesocolumnar firings. This argument analytically articulates the meaning of "neuronal degeneracy" and also of the ubiquitous, often ambiguous "average neuron." However, reasonable properties of mesocolumns, not of average neurons, are developed here for macroscopic study.

The properly calculated distribution contains nearest-neighbor mesocolumnar interactions expressed as derivative correction terms. Eq. (3.20) verifies that in macroscopic activity, where mesocolumnar (patterns of) firing vary smoothly over neighboring mesocolumns, it is consistent to approximate mesocolumnar interactions by including only second-order gradient correction terms. Sec. 4 calculates macroscopic states of mesocolumnar firings, which are subject to these constraints of Eq. (3.20). Excitatory and inhibitory sensitivity to the neuronal parameters survives, similar to the sensitivity encountered by single neurons.

In this manner, microscopic degrees of freedom of many types of neurons (many of which are only crudely classified by the above definitions), synapses, neurotransmitters, cellular architecture, and circuitries, may be practically weighted and averaged for macroscopic considerations.

3.1. *Mesocolumns*

Define a mesocolumn as a domain with N neurons, with stochastic memory $\tau \approx$ one to several τ_n. Denote by indices E and I two chemically and anatomically independent firing fields; G denotes either field.

For this study do not consider dynamic synaptic modifications, which typically take place as a result of one to many epochs of macroscopic temporal activity effecting such plastic changes [42]. Therefore, take as independent of space-time:

$$|v_{jk}|,\ \phi_{jk} \rightarrow v^G,\ \phi^G \tag{3.1}$$

$$V_j,\ A_{jk},\ B_{jk} \to V^G,\ A^G,\ B^G$$

$$\tfrac{1}{2}\,A^G + B^G = a^G\ .$$

The greater importance of I synapses (circuitry, proximity to soma) increases effective inhibitory v_{jk} and a_{jk}. Take

$$\sum_k v_{jk} A_{jk} \approx v^G A^G (N^E - N^I),\ N = N^E + N^I\ . \tag{3.2}$$

Write p_{σ_j} as the response of neuron j to its surrounding mesocolumn, represented by

$$F_j^G = \beta^G(\gamma^G - \alpha^G \not{M}_j)/(1 + \alpha^G M_j)^{\frac{1}{2}}\ , \tag{3.3a}$$

$$\not{M}_j = (M_j^E - M_j^I) = (\sum_{k,E} \sigma_k - \sum_{k,I} \sigma_k)\ ,\ M_j^G = N m_j^G\ ,$$

$$M_j = (M_j^E + M_j^I)\ ,\ -N^G \le M_j^G \le N^G\ ,$$

where the six mesoscopic parameters are defined by:

$$\alpha^G = A^G/(2Na^G) \ll 1 \tag{3.3b}$$

$$\beta^G = [Na^G(1 + (\phi^G/v^G)^2)^{-1}\pi^{-1}]^{\frac{1}{2}} < N^{\frac{1}{2}}$$

$$\gamma^G = V^G/(a^G v^G N) - (N^E - N^I)/N.$$

Typically [39],

$$\not{m}_j^G < \pm 10^{-1} \tag{3.4}$$

making p_{σ_j} sensitive to $F_j^G \approx 0$ for small variations of $\not{M}_j$ about small γ^G. $|F_j^G| < \pi/2$ permits the asymptotic approximation in Sec. 2.

For future studies in Secs. 4 and 5, it is useful to perform the valid expansion:

$$(1 + \alpha^G M_j)^{-\frac{1}{2}} \approx 1 - \alpha^G M_j/2 + 3(\alpha^G M_j)^2/8 - + \cdots \tag{3.5}$$

Note that Eq. (3.3) may be generalized to include additional $E - I$ structure without affecting the following development. For example,

$$\alpha^G \not{M}_j,\ \alpha^G M_j \to \sum_{G'} \alpha^{GG'} M_j^{G'},\ \sum_{G'} \alpha'^{GG'} M_j^{G'}. \tag{3.6}$$

3.2. *Nearest-Neighbor (NN) Interactions*

Define $Nn^G(r)$, a mesocolumn centered at the 2-dimensional point r, as the mesocolumnar average of σ_j, as in Eq. (3.3). (See Figs. 1b-b'.) Derive the conditional probability $p(Nn^G|M^{\bar G})$ for the firing transition to mesocolumn $Nn^G(r)$ after time τ, from all contributing mesocolumns $Nm^{\bar G}(r')$ associated with neurons σ_k. $M^{\bar G}$ represents contributions from both M^E and M^I in F_j^G:

$$\prod_G p(Nn^G|M^{\bar G}) = \sum_{\sigma_j = \pm 1} \delta(\sum_{j,E} \sigma_j - Nn^E)\delta(\sum_{j,I} \sigma_j - Nn^I) \prod_j^N p_{\sigma_j}\ . \tag{3.7}$$

For $r = r'$, the result of the mesoscopic averaging of independent E and I fields is easily intuited, as explained above: The contribution of N^G averaged ±1 firings to establish a firing Nn^G has degeneracy

$$\begin{bmatrix} N^G \\ \frac{1}{2}(Nn^G + N^G) \end{bmatrix}, \; - N^G \le Nn^G \le N^G \ . \tag{3.8a}$$

A binomial distribution of Nn^G is therefore anticipated, weighted by the averaged firing probability, with

$$\{F_j^G\} \to \{\text{typical } F^G\} \tag{3.8b}$$

from Eq. (2.19). However, the following analysis also permits NN interactions to be calculated.

Calculate Eq. (3.7):

$$p(Nn^G|M^{\bar{G}}) = (2\pi)^{-1} \int_{-\infty}^{\infty} dQ^G \exp(iNn^G Q^G) \prod_j^{N^G} C_j^G \ , \tag{3.9}$$

$$C_j^G\{F_j^G[M^{\bar{G}}(r + \varepsilon)]\} = \cosh(F_j^G + iQ^G)\text{sech}F_j^G, \;\; r + \varepsilon = r' \ .$$

Expand C_j^G about $Nn^G(r)$, using directional derivatives $D_{\hat{\varepsilon}}^{1,2}$,

$$C_j^G \approx [1 + |\varepsilon| D_{\hat{\varepsilon}}^1 + \frac{1}{2}|\varepsilon|^2 D_{\hat{\varepsilon}}^2] C^G(r), \tag{3.10}$$

$$\hat{\varepsilon} = \varepsilon/|\varepsilon| = (r' - r)/|r' - r| \ ,$$

retaining only first and second order derivatives of M and $\not{M}$. Obtain

$$p(Nn^G|M^{\bar{G}}) = (2\pi)^{-1} \int dQ^G \exp(iNn^G Q^G)[\cosh(F^G + iQ^G)\text{sech}F^G]^{N^G} \tag{3.11}$$

$$\times \prod_j^{N^G} (1 + dF_{j1}^G K_1 + dF_{j2}^G K_2),$$

$$K_1 = \sinh(iQ^G)\text{sech}(F^G + iQ^G)\text{sech}F^G \ ,$$

$$K_2 = - 2K_1 \tanh F^G \ ,$$

where dF_{j1}^G and dF_{j2}^G are calculated below.

To zero order, neglect K_1 and K_2 terms, express $\cosh^{N^G}$ as a sum of exponentials, binomial-expand, do $\int dQ^G$, and obtain

$$p_0(Nn^G|M^{\bar{G}}) = (1 + f^G)^{-N^G} \binom{N^G}{\lambda^G} (f^G)^{\lambda^G}, \tag{3.12}$$

$$f^G = \exp(-2F^G), \; \lambda^G = [[\frac{1}{2}(Nn^G + N^G)]],$$

where λ^G is the greatest integer in the double brackets on RHS. This is the anticipated binomial distribution with mean

$$< \lambda^G >_0 = N^G \exp(-F^G)/[(\exp(F^G) + \exp(-F^G)], \tag{3.13}$$

$$< Nn^G >_0 = - N^G \tanh F^G \ ,$$

and variance

$$< \lambda^G \lambda^{G'} >_0 - < \lambda^G >_0 < \lambda^{G'} >_0 = {}^1\!/_4 \delta_{G'}^G N^G \text{sech}^2 F^G \ . \tag{3.14}$$

Compare Eq. (3.13) with Eq. (2.19). Thus n^G and m^G are defined as mesocolumnar averaged neurons, with their anticipated zero order statistical firing weights. Eq. (3.13) explicitly demonstrates how sensitive $< Nn^G >_0$ is to changes in sign of the threshold factor F^G in the step-function $\tanh F^G$: $\tanh(\pm\infty) = \pm 1$, $\tanh(\pm 1) = \pm .76$, $\tanh(0) = 0$. As expected, in the absence of interactions, setting $B^G = 0$ and with $Nm^G = - N^G$, then $< Nn^G >_0 = - N^G$; no firing occurs. As $Nm^G \ge \gamma^G / \alpha^G$, so $< Nn^G >_0 \ge 0$ maintains $< Nn^G >_0 > - N^G$ for all Nm^G, and thus B^G may be taken to simulate nonsynaptic influences on Nn^G. Other long-ranged synaptic influences are included in Sec. 5.

To define NN interactions, calculate the differential terms in Eq. (3.11):

$$\prod_j^{N^G} (1 + dF_{j1}^G K_1 + dF_{j2}^G K_2) \approx [1 + \sum_j^{N^G} (dF_{j1}^G K_1 + dF_{j2}^G K_2)] \tag{3.15}$$

$$\equiv 1 + dF_1^G K_1 + dF_2^G K_2 \ ,$$

defining dF_1^G and dF_2^G. This is validated by Eq. (3.20) below. To calculate dF_1^G and dF_2^G, define $\rho < 10^2 \mu$m as the spatial extent of a mesocolumn. ρ is fixed by the value of N.

$$\sum_j \to N^G \rho^{-2} \int_{x - \frac{1}{2}\rho}^{x + \frac{1}{2}\rho} dx' \int_{y - \frac{1}{2}\rho}^{y + \frac{1}{2}\rho} dy' , \ \ (x, y) = r . \tag{3.16}$$

Obtain:

$$dF_1^G = 1.24 N^G (\rho^2/24) \alpha^G (1 + \alpha^G M)^{-\frac{1}{2}} \tag{3.17}$$

$$\times\{ - \tfrac{1}{2} F^G (1 + \alpha^G M)^{-\frac{1}{2}} (\nabla^2 M) - \beta^G (\nabla^2 \not{M})$$

$$+ \alpha^G (1 + \alpha^G M)^{-1} (\nabla M) \cdot [\beta^G (\nabla \not{M}) + \tfrac{3}{4} F^G (1 + \alpha^G M)^{-\frac{1}{2}} (\nabla M)]\}$$

$$dF_2^G = 0.513 N^G (\rho^2/24) (\alpha^G)^2 (1 + \alpha^G M)^{-1} [\beta^G (\nabla \not{M}) + \tfrac{1}{2} F^G (1 + \alpha^G M)^{-\frac{1}{2}} (\nabla M)]^2 \ .$$

Calculate K_1 and K_2 using the same procedures giving Eq. (3.12). The conditional probability for average mesocolumns $M^{\bar{G}} \to Nn^G$ within τ, including NN, is:

$$p(Nn^G | M^{\bar{G}}) = [p_0 + dF_1^G p_1 + dF_2^G p_2] , \tag{3.18}$$

$$p_1(Nn^G | M^{\bar{G}}) = (1 + f^G)^{-N^G - 1} \binom{N^G - 1}{\lambda^G} (f^G)^{\lambda^G} [f^G (1 - \delta_{Nn^G, N^G}) - (1 - \delta_{Nn^G, -N^G})]$$

$$\approx - \tanh F^G p_0(Nn^G | M^{\bar{G}}) ,$$

$$p_2(Nn^G | M^{\bar{G}}) = - 2 p_1 \tanh F^G \ .$$

This can be rewritten as:

$$p(Nn^G | M^{\bar{G}}) = (1 + dF^G) p_0(Nn^G | M^{\bar{G}}) , \tag{3.19}$$

$$dF^G = - \tanh F^G (dF_1^G - 2 dF_2^G \tanh F^G) .$$

The sign and magnitude of contributions to dF^G can vary from repulsive to attractive interactions, depending on the values of the mesocolumnar parameters and the M^G firings as they influence F^G. (Strict averaging of $(\nabla M^E) \cdot (\nabla M^I)$ over independent E and I firings gives zero contribution. However, these terms are carried along here to demonstrate how anisotropic mesocolumnar interactions might be studied.)

Consistency of the above scheme requires a definition of this long-wavelength scale. The conditions placed on Eq. (3.17) for Eqs. (3.15) and (3.18) to be valid are evaluated to be essentially

$$1 > (\rho \nabla M^G)^2/(24N), \tag{3.20}$$

which is consistent with this macroscopic development. Calculations in Sec. 5 confirm the consistency of this derivation of inter-columnar interactions with empirical observations.

4. Macroscopic Regions

The prototypical diffusion system describes Brownian motion wherein the stochastic memory of the macroscopic system depends only on the immediate past history of the system one specified unit of time previous, and in a linearly functional manner [40,41,84]. Here, the G-space diffusion is expressed by a nonlinear dependence on this past firing state, and the stochastic memory must be carefully defined. Real-space diffusion is represented by the gradient couplings. These fluctuations are physically important for various excitations and possible critical behavior of second order phase transitions between ordered and disordered states [45,101,117]. It should be noted that some investigators have been unwilling to accept the analogy between equilibrium and nonequilibrium long-ranged order as arising from spontaneous symmetry breaking. However, recent research demonstrates that this analogy is indeed often appropriate [99].

Figs. 1c and 1c’ illustrate how the mesocolumnar structure of Sec. 3 is a substrate for activity persisting for hundreds of msec over a spatial region containing $\Lambda \approx 5 \times 10^5$ mesocolumns, spanning $\approx 10^{-2}$ of a total cortical area of $4 \times 10^{11} \mu\text{m}^2$. Extended regional activity is possible, whereby conglomerates of 10 to 30 regions may interact [81].

Sec. 3 has calculated the mesocolumnar conditional probability that a given mesocolumn will fire, given its direct interactions with other mesocolumns just previously firing. Thus a transition rate from one state of mesocolumnar firing to another state just afterwards is obtained. A string, or path of these conditional probabilities connects the mesocolumnar firings at one time to the firing at any time afterwards. Many paths may link the same initial and final state. In this way the long-time conditional probability of all possible mesocolumnar firings at any given time is obtained. A Lagrangian is thereby derived which explicitly describes the time evolution of the neocortical region in terms of its initial distribution of firings, and expressed in terms of its mesoscopic order parameters which retain a functional form derived from microscopic neuronal interactions. (See Fig. 2.) A major benefit derived from this formalism is a variational principle that permits extrema equations to be developed.

This Lagrangian can be expanded into a simple fourth order polynomial of powers of the mesocolumnar firings, yielding a generalized Ginzburg-Landau (GL) expression [41]. At the present stage of development of statistical mechanics, for many purposes this simple form is a practical necessity to continue future studies. It is shown here that this expansion is valid for the neocortical system. This also makes it possible to draw analogies to the "orienting field" and "temperature" of other collective systems. Insofar as GL expressions can also be derived from Ising-type systems, albeit with much approximation [117], then with analytic hindsight some analogies may yet be correctly drawn between neocortical and magnetic systems [18]. Typically, in Ising-type systems, intuited GL models have predicted and described collective systems quite well, so that GL theory there is aptly considered a good physical approach. Neocortical studies have not yet achieved such a wealth of data or formalism, and so the existence of a GL expression as derived here is nontrivial. Conversely, similar analytic development of a GL expression from relatively microscopic principles is a luxury afforded to only a few physical systems.

4.1. *Mesocolumnar Short-Time Propagator*

To first order in dF, the distribution p of Eq. (3.18) can be defined in terms of variables that facilitate this development. For large N^G and $N^G F^G$, this binomial distribution is asymptotically Gaussian

[70].

$$p(Nn^G|M^{\bar{G}}) = (2\pi\tau g'^{GG})^{\frac{1}{2}} \exp(-N\tau \underline{L}^G), \tag{4.1}$$

$$\underline{L}^G = N^{-1}[(Nn^G - M^G)/\tau - g'^G]^2/(2g'^{GG}),$$

$$g'^G = -\,\tau^{-1}[M^G + N^G(1 + dF^G)\tanh F^G],$$

$$g'^{GG} = \tau^{-1} N^G(1 + dF^G)\text{sech}^2 F^G \ .$$

Define time epochs t in units of τ, in terms of integer $s \ge 0$, from an initial time t_0:

$$t_s = s\tau + t_0 \ . \tag{4.2}$$

For the large time epochs to be considered, a continuum differential time scale is defined by $dt \le \tau$. Within τ, consistent with this long-time development and consistent with the previous mesocolumnar averaging of neuronal interactions, define

$$\underline{L}^G = N^{-1}(\dot{M}^G - g'^G)^2/(2g'^{GG}), \tag{4.3}$$

$$\tau\dot{M}^G = \tau dM^G/dt = Nn^G - M^G \ .$$

Then the Markovian mesocolumnar short-time propagator, the conditional probability p, is developed for short times $\theta \approx \tau$ relative to any fixed time t:

$$p[M(t+\theta), t+\theta] = (2\pi\theta)^{-1} \int dM(t)\ g'(t)^{\frac{1}{2}} \tag{4.4}$$

$$\times \exp\{-N\underline{S}[M(t+\theta), M(t)]\}\ p[M(t), t],$$

$$dM = dM^E dM^I \ ,$$

$$g' = (det\ g')^{-1} = (g'^{EE} g'^{II})^{-1} \ ,$$

and $\underline{S}$ is defined by requiring that the system evolve by the principle of maximal probability:

$$\underline{S} = \min \int_t^{t+\theta} dt'\ \underline{L}[\dot{M}(t'), M(t')], \tag{4.5}$$

$$\underline{L} = \underline{L}^E + \underline{L}^I \ .$$

For small θ, relative to the long times considered, with $N \gg 1$, contributions to p at $t+\theta$ are heavily weighted within

$$|\Delta M^G| = |M^G(t+\theta) - M^G(t)| < \theta^{\frac{1}{2}} \ , \tag{4.6}$$

and therefore the quadratic $\dot{M}$ terms in $\underline{L}$ must be carefully developed.

4.2. *Regional Short-Time Propagator*

Define the Λ-dimensional vector $\tilde{M}_s$ at time t_s:

$$\tilde{M}_s = \{M_s^\nu = M_s(r^\nu);\ \nu = 1, \cdots, \Lambda\}, \tag{4.7}$$

$$M_s^\nu = \{M_s^{G\nu};\ G = E, I\}.$$

For macroscopic space-time considerations, mesoscopic ρ and τ scales are measured by dr and dt. In the continuum limits of r and t:

$$M_s^{G\nu} \to M^G(r,t),\ \dot{M}_s^{G\nu} \to dM^G/dt, \tag{4.8}$$

$$(M^{G,\nu+1} - M^{G\nu})/(r^{\nu+1} - r^\nu) \to \nabla_r M^G\ .$$

The previous development of mesocolumnar interactions via NN derivative couplings permits the regional short-time propagator $\tilde{p}$ to be developed in terms of the Lagrangian $\underline{L}$ [28]:

$$\tilde{p}(\tilde{M}) = (2\pi\theta)^{-\Lambda/2} \int d\tilde{M}\, g'^{\Lambda/2} \exp[-N\tilde{S}(\tilde{M})]\tilde{p}(\tilde{M}), \tag{4.9a}$$

$$\tilde{S} = \min \int_t^{t+\theta} dt' L[\dot{M}(t'), M(t')],$$

$$L = \Lambda\Omega^{-1} \int d^2r\ \underline{L}\ ,$$

where Ω is the area of the region considered, and

$$\Lambda\Omega^{-1} \int d^2r = \Lambda\Omega^{-1} \int dxdy = \lim_{\substack{\rho\to 0\\ \Lambda\to\infty}} \sum_{\nu=1}^{\Lambda}\ . \tag{4.9b}$$

The Euler-Lagrange (EL) equations, giving the extrema $\ll M^G \gg$, are obtained from $\delta\underline{\tilde{S}}=0$ [11]. The Einstein convention is used to designate summation over repeated indices, and the following notation for derivatives is used:

$$(\cdots)_{:z} = d(\cdots)/dz,\ \ z = \{x, y\}, \tag{4.10}$$

$$(\cdots)_{,G} = \partial(\cdots)/\partial M^G,\ \ (\cdots)_{,\dot{G}} = \partial(\cdots)/\partial(dM^G/dt),$$

$$(\cdots)_{,G_{:z}} = \partial(\cdots)/\partial(dM^G/dz),$$

$$(\cdots)_{,\nabla G} = \hat{x}\partial(\cdots)/\partial(dM^G/dx) + \hat{y}\partial(\cdots)/\partial(dM^G/dy).$$

The EL equations are:

$$\delta\underline{L} = 0, \tag{4.11}$$

$$\delta_G\underline{L} = \underline{L}_{,G} - \nabla\cdot\underline{L}_{,\nabla G} - \underline{L}_{,\dot{G}:t} = 0,$$

$$\nabla\cdot\underline{L}_{,\nabla G} = \underline{L}_{,G_{:z}:z}$$

$$= (\underline{L}_{,G_{:z}}{}_{,G'})M^{G'}{}_{:z} + (\underline{L}_{,G_{:z}}{}_{,G'_{:z}})M^{G'}{}_{:zz}$$

$$\underline{L}_{,\dot{G}:t} = (\underline{L}_{,\dot{G}}{}_{,G'})\dot{M}^{G'} + (\underline{L}_{,\dot{G}}{}_{,\dot{G}'})\ddot{M}^{G'}.$$

This exhibits the extremum condition as a set of 12 first-order differential equations in the 12 variables $\{M^G, \dot{M}^G, \ddot{M}^G, M^G{}_{:z}, M^G{}_{:zz}\}$ in $r - t = (x, y, t)$ space, with coefficients nonlinear in M^G.

To facilitate further development, consistent with previous approximations, expand $\underline{L}$ to first order in dF, dropping third and higher order terms, such as $\dot{M}dF$, and integrate terms in $M^G{}_{:zz}$ by parts to yield quadratic terms in in $M^G{}_{:z}$. In this first study, drop the boundary terms. Obtain (before integrating by parts):

$$\underline{L} = \underline{T} - \underline{V}' \ , \ \ \underline{V}' = -\,(2N\tau)^{-1} g_{G'G} g^G (g^G + 2M^G/\tau) dF^G \ , \tag{4.12}$$

$$(\cdots)\nabla^2 M^G \rightarrow -\,(\nabla \cdots) \cdot (\nabla M^G) = -\,(\cdots)_{,G'}\, M^{G'}{}_{:z} M^G{}_{:z},$$

where g^G and $g^{GG'}$ are defined by setting the dF terms in g'^G and $g'^{GG'}$ to zero, and $g_{GG'} \equiv (g^{GG'})^{-1}$.

4.3. *Regional Long-Time Propagator*

With $\tilde{p}$ properly defined by this space-time mesh, a path-integral formulation for the regional long-time propagator $\tilde{P}$ at time $t = (u+1)\theta + t_0$ is developed:

$$\tilde{P}[\tilde{M}(t)]\, d\tilde{M}(t) = \int \cdots \int \underline{D}\tilde{M} \exp(-N \int_{t_0}^{t} dt' L), \tag{4.13}$$

$$\tilde{P}[\tilde{M}(t_0)] = \delta(\tilde{M} - \tilde{M}_0),$$

$$\underline{D}\tilde{M} = \prod_{s=1}^{u+1} \prod_{\nu}^{\Lambda} \prod_{G}^{E,I} (2\pi\theta)^{-\frac{1}{2}} (g_s^\nu)^{1/4} dM_s^{G\nu}.$$

Note that even for $N\tau\underline{L} \approx 1$, $N \int_{t_0}^{t} dt' \, \underline{L}$ is very large for macroscopically large time $(t - t_0)$ and size Λ, demonstrating how extrema of $\underline{L}$ define peaked maximum probability states. Eq. (4.13) contains the dynamics of macroscopic causal irreversibility, whereby $\tilde{P}$ is an unstable fixed point about which deviations from the extremum are greatly amplified [21].

4.4. *Information, Potential, and Long-Ranged Interactions*

With reference to a steady state $\bar{P}(\tilde{M})$, when it exists, an analytic definition of the information gain $\hat{\Upsilon}$ in state $\tilde{P}'(\tilde{M})$ is defined by [38,41]

$$\hat{\Upsilon}[\tilde{P}'] = \int \cdots \int \underline{D}\tilde{M} \ \tilde{P}' \ln(\tilde{P}'/\bar{P}). \tag{4.14}$$

Although $\hat{\Upsilon}$ is well defined and useful for discussing macroscopic neocortical activity, it may not be as useful for all applications. Certainly many important local changes of information effected by the neocortical system are a function of the microscopic degrees of freedom already averaged over for the purposes of this study. However, it should also be noted that the path integral in Eq. (4.13) represents an enormous number of spatial-temporal degrees of freedom of the mesoscopic system.

The minimization of $\hat{\Upsilon}$ with respect to $\tilde{P}'$, with $\tilde{M}$ constrained to its (possibly multi-valued) mean trajectory $< \tilde{M} >'$, is formalized by the use of Lagrange parameters J_G. This results in the Legendre transform of $\ln \tilde{P}$ [33], and is equivalent to the generating functional Υ defined in the presence of extrinsic sources J_G [1,35,38]. These sources specify firing constraints imposed on a given region of mesocolumns from long-ranged extrinsic or inter-regional afferents, e.g. from ipsilateral association, contralateral commissural, and thalamocortical processes.

$$\hat{\Upsilon}(\min\{\tilde{P}[< \tilde{M} >_J]\}) = \Upsilon[< \tilde{M} >_J] \tag{4.15a}$$

$$= -\ln \tilde{P}(J) + \int dt \int d^2 r < \tilde{M}^G >_J J_G \ ,$$

where $\tilde{P}(J)$ is calculated by replacing $\underline{L}$ by $\underline{L}^J$:

$$\underline{L}^J = \underline{L} + M^G J_G/(2N\tau). \tag{4.15b}$$

If J is distributed, then $\underline{D}J$ measures effects on $\underline{L}^J$. A Hamiltonian representation, including $J_G \dot{M}^G$ interactions, is also readily derived [1]. This may be useful for describing long-ranged constraints that directly affect rates of change of columnar firings.

$\Upsilon_{J=0}$ is a proper potential, with free-energy like properties, having a true minimum about $(< \tilde{M} >_J - \ll \tilde{M} \gg)$, where $\ll \tilde{M} \gg$ is the extremum of $\underline{L}$ obtained by maximizing $\tilde{P}(\tilde{M})$ [5]. Its lowest order approximate $\Upsilon^{(0)}$ gives the mean field approximation:

$$\Upsilon^{(0)} = \underline{\tilde{S}}(< \tilde{M} >). \tag{4.16}$$

In the context of an additional internal energy term, *energy*/($k_B \times$ *temperature*), where k_B is the Boltzmann constant, $-N\Upsilon^{(0)}$ contributes *entropy*/k_B to the "free energy" expression. If the internal energy is proportional to M^G, up to a constant absorbed by the normalization of $\tilde{P}[J]$, then the J_G term contribution to $N\Upsilon_J$ may be considered to include the temperature-dependent energy term. Of course, these thermodynamic correlations to the statistical mechanical results derived here are essentially mnemonic aids. For example, the "*entropy*" is a function of the mesoscopic parameters, which ultimately are functions of microscopic chemical reactions, which have temperature-dependent reaction rates.

Define uniform $\bar{M}^G$ as static, r-space averaged M^G:

$$\dot{\bar{M}}^G = 0 = (\nabla \bar{M}^G)^2 \ . \tag{4.17}$$

Then extrema $\ll \bar{M} \gg$ can be simply calculated as

$$\partial \underline{L} / \partial \bar{M}^G = 0. \tag{4.18}$$

The potential surface $\Upsilon^{(0)}$, mapped out over $\bar{M}^E - \bar{M}^I$ coordinates, provides a useful picture of the neocortical system.

As seen from Eq. (4.1), $\underline{T}$, defined in Eq. (4.12) as the dF-independent part of $\underline{L}$, is scale-independent of N. Therefore, the small scale of the neocortical system, about which the system fluctuates, is derived to be N^{-1}, the inverse of the number of neurons in a mesocolumn. This is interpreted as the effective "temperature" or inherent noise of the GL system. The sharpness of the tanh F^G step-function contribution to the mean firing is sensitive to a factor of $N^{\frac{1}{2}}$ in β^G in F^G. Additionally, the strength of coupling between mesocolumns scales as $N^{3/2}$. Thus the neuronal size of mesocolumns directly affects the breadth and depth of the information processing capability of neocortex. It is interesting to note that human visual cortex, which may be assumed to require the finest tuning in neocortex, is unique in having twice the number of neurons per minicolumn than other regions of neocortex [81].

Effects of fluctuations begin to appear in the next order of the N^{-1} cumulant expansion of Υ [5,14]. An important quantity is the space-time correlation function Φ:

$$\begin{aligned} \Phi^{GG'}(r - r'; t - t') &= < M^G(r,t) M^{G'}(r',t') > - < M^G(r,t) >< M^{G'}(r',t') > \\ &= \delta^2 (\ln \tilde{P}[J]) / \delta J_G \delta J_{G'} \Big|_{J=0} \\ &= [\delta^2 \Upsilon / \delta < M^G >_J \ \delta < M^{G'} >_J]^{-1} \Big|_{J=0} \ , \end{aligned} \tag{4.19}$$

where the last equality holds for all J. Weak fluctuations may also be treated by WKB-type expansions of $\tilde{P}$ in orders of N^{-1}. In regions of second order phase transitions, long-ranged and long-time correlations in Φ can appear in the macroscopic limit [69]. Typically, these are the object of renormalization group (RG) studies.

4.5. *Ginzburg-Landau (GL) Lagrangian*

To facilitate further study of the long-time macroscopic propagator defined by Eqs. (4.12) and (4.13), put $\tau^{-2} g_s^\nu$ from $\underline{D}\tilde{M}$ into $\underline{L}$, defining an effective $\underline{L}_e$:

$$\underline{L}_e = \underline{L} + (N\tau)^{-1} \ln(\tau g^{-\frac{1}{2}}) \approx [1 + O(N^{-1} \ln N)]\underline{L} \ . \tag{4.20}$$

Expand all functions in $\underline{L}_e$ to order $(\nabla M^G)^2$ and in powers of M^G. This is permitted by Eqs. (2.18) and (3.5). A generalized GL Lagrangian density in "flat" M^G-space is obtained from $\underline{L}_e$ (or similarly from $\underline{L}$) of the form

$$\underline{L}_{GL} = \sum_{i,i'} \sum_{j,j'} \sum_{k,k'} b_{ii'jj'kk'} \tag{4.21}$$

$$\times(\underline{M}^G)^k(\underline{M}^{G'})^{k'}(\nabla\underline{M}^G)^j(\nabla\underline{M}^{G'})^{j'}(\dot{\underline{M}}^G)^i(\dot{\underline{M}}^{G'})^{i'} \ , \ \underline{M}^G = M^G - \ll M^G \gg \ ,$$

where the b’s are the expansion coefficients, expressed in terms of α^G, β^G, andγ^G of Eq. (3.3). Empirical values of these parameters ensure that the most important contributions come from small values of k and k'. Typically, only terms up to the quadratic power in mixed derivative and static terms, and up to the quartic power in solely static M^G terms, are kept in this expansion.

4.6. *Differential Propagator — Reduction to Other Studies*

This subsection is not essential to the development of Secs. 4 and 5. However, comparison of this formulation to other analytic treatments of cortex is made straightforwardly by examining the differential equations satisfied by $\tilde{p}[\tilde{M}(t+\theta)]$ in Eq. (4.9). The EL equations derived by the principle of maximum probability, Eq. (4.5), are used to express $\Delta M^G = M^G(t+\theta) - M^G(t)$ as a power series in ΔM^G, which become the variables of integration. Only terms quadratic in ΔM^G are kept in $\tilde{\underline{S}}$. All other terms, as well as g and $\tilde{p}[\tilde{M}(t)]$ are expanded in a power series in ΔM^G. To order θ and to second order in derivatives in $\tilde{p}$ with respect to M^G, an expansion up to order $(M^G)^6$ is required [16,65]. The resulting integrals are simply moments of Gaussian distributions. In terms of the "drift" H^G and the "diffusion" $g^{GG'}$ (in G-space):

$$\dot{\tilde{p}} = \Omega^{-1} \int d^2r[(-H^G + \tfrac{1}{2}(g^{GG'})_{,G'} - N\underline{V}] \ , \tag{4.22}$$

$$\tilde{p}[\tilde{M}(t_0)] = \delta(\tilde{M} - \tilde{M}_0),$$

$$H^G = g^G + \tfrac{1}{2} g^{-\frac{1}{2}}(g^{\frac{1}{2}} g^{GG'})_{,G'}$$

$$\underline{V} = \underline{V}' - (\tfrac{1}{2} g^G{}_{;G} + R/6)/N$$

$$g^G{}_{;G} = g^G{}_{,G} + \Gamma^F_{JF} g^G = g^{-\frac{1}{2}}(g^{\frac{1}{2}} g^G)_{,G}$$

$$\Gamma^F_{JK} \equiv g^{LF}[JK, L] = g^{LF}(g_{JL,K} + g_{KL,J} - g_{JK,L}),$$

$$g_{GG'} = (g^{GG'})^{-1} \ ,$$

$$R = g^{JL} R_{JL} = g^{JL} g^{FK} R_{FJKL} \ ,$$

$$R_{FJKL} = \tfrac{1}{2}(g_{FK,JL} - g_{JK,FL} - g_{FL,JK} + g_{JL,FK})$$

$$+ g_{MN}(\Gamma^M_{FK}\Gamma^N_{JL} - \Gamma^M_{FL}\Gamma^N_{JK}).$$

In the literature, this calculation is done in the opposite order. That is, starting with the differential equation, typically the Fokker-Planck equation ($\underline{V} = 0$), the short-time propagator is derived and further developed for long times [21-25,34-38,63-66]. Then the terms $\frac{1}{2} g^G{}_{;G} + R/6$ must be added to $\underline{L}$ in order that $\tilde{p}$ satisfy the original differential equation. (This study has taken Eqs. (4.1) and (4.3) to define an explicit non-covariant $\underline{L}$.) Care must be taken with the time discretization employed [63]. Some investigators also use other constraints, such as developing a WKB approximation, instead of the principle of maximum probability, yielding a contribution to $\underline{V}$ of $R/12$ instead of $R/6$ [38]. $R/8$ is obtained using other principles that include fully stochastic paths at arbitrarily short time intervals [23-25]. In this study, the biophysics dictates the appropriate discretization and probability constraints. The magnitude of this contribution, calculated in Sec.5, confirms that neocortex is the first system investigated in which this term has measurable physical consequences [24].

All these calculations are facilitated by noting that the system is defined in a Riemannian geometry with metric tensor $g^{GG'}$ and and a contravariant (upper indices) and covariant (lower indices) tensor algebra. Γ^F_{JK} is the affine connection, R the Riemann curvature scalar, R_{JL} the Ricci tensor, and R_{FJKL} the Riemann-Christofell tensor [110]:

$$R^{F}{}_{JKL} Z_F = Z_{J;LK} - Z_{J;KL} \ , \tag{4.23}$$

Z an arbitrary covariant vector. Numerical calculations described in Sec. 5 demonstrate that R and $g^G{}_{;G}$ for neocortex are non-negligible contributions to the differential propagator. With $g^{GG'}$ diagonal for neocortex,

$$\begin{aligned} R = {} & g^{-1}(g_{EE},_{II} + g_{II},_{EE}) \\ & - \tfrac{1}{2} g^{-2}\{g_{II}[g_{EE},_E\, g_{II},_E + (g_{EE},_I)^2] + g_{EE}[g_{II},_I\, g_{EE},_I + (g_{II},_E)^2]\}. \end{aligned} \tag{4.24}$$

In this system, a nonzero interaction potential exists from nonlinear g^{GG} and dF contributions, and a Schroedinger-type equation (with $i\hbar \rightarrow N^{-1}$, where $\hbar$ is Planck's constant) is derived instead of a Fokker-Planck equation. Other studies using phenomenological Fokker-Planck equations for neocortex, or an equivalent set of Langevin rate equations [64,111], do not properly include these nonlinear contributions. If this objection is arbitrarily disregarded; if functions $\hat{g}^G_j$ spanning both microscopic and mesoscopic systems can be defined such that $g^{GG'} = \hat{g}^G_j \hat{g}^{G'}_j$; and if functions η_j are taken to represent microscopic Gaussian white noise; then a Langevin representation of Eq. (4.22) is also possible:

$$\begin{aligned} & \dot{M}^G = g^G + \hat{g}^G_j \eta_j \ , \\ & < \eta_j(t) >_\eta = 0, \\ & < \eta_j(t)\eta_{j'}(t') >_\eta = \delta_{jj'}\delta(t - t'), \end{aligned} \tag{4.25}$$

If dF terms are included to express mesocolumnar coupling, the resulting Langevin equations specify a reaction-diffusion system in real-space and time coordinates. g^G and $g^{GG'}$ are defined in this context as the first and second rate-jump-moments of this $\underline{V} = 0$ system.

If all the $\underline{V}$ and $g^{GG'}$ are arbitrarily set to zero, the resulting equations, written in terms of "average neurons," are just those taken as the starting point for the phenomenological pioneering modelling of cortex in previous studies [30,31,115,116], upon which other studies referenced in Sec. 1.2 are based. In some studies, additive white noise, which assumes $\hat{g}^G_j$ = constant, is included, and then typically several thousand computer "trials" are examined to find most probable trajectories. In Secs. 4 and 5, a variational principle is derived to directly calculate these extrema trajectories, without requiring the above assumptions.

4.7. *Further Application of Other Formalisms*

This subsection also is not essential to the development of Secs. 4 and 5. However, this study permits other related issues to be discussed that have been raised by other investigators of neocortical modelling.

Renormalization group (RG) analyses may be appropriate to the GL polynomial obtained. This nonequilibrium system has well defined: diffusion, short-ranged interactions, derivative NN couplings, continuous order parameters, 2 spatial dimensions, and 2 independent fields. Time-dependent RG field theories can be applied to Eqs. (4.12), (4.15) and (4.19) which rely on older RG techniques [11,14,19,20]. However, in two dimensions renormalization of all b-coefficients of $\underline{L}$ may be necessary. Real-space techniques developed primarily for 2-dimensional systems may be more practical [71-74,85], using Eq. (4.22). This latter approach shows promise of eventually consistently treating both first and second order phase transitions, if more calculational accuracy can be achieved [75].

This system may also support various collective phenomena described by phase transitions of E and I (or $E \pm I$, etc.) fields considered separately, by extrinsically, chemically or electrically, maintaining the other field fixed. Then the linear b-coefficient would be analogous to the "orienting field" of a magnetic system, the quadratic b-coefficient would be proportional to the relative "critical temperature" [69].

It should be noted that although M^G-space anisotropies are present, the possibility exists of transforming away $(M^G)^3$ terms, effectively rounding out a first order phase transition [69]. Other long-ranged long-time interactions considered in Eq. (4.15b) may act to drive the linear coefficient to zero, permitting symmetric bifurcation, and the onset of a second order phase transition must be considered. Thus a plausible hypothesis is presented, whereby long-ranged afferents act to drive the system of short-ranged interactions through phase transitions. An example of this mechanism is further discussed in Sec. 5.

The mesocolumnar parameters, initial conditions, and long-range source terms may be taken as dependent on different regions. Long-range interactions between regions I and II in Fig. 1c’ may be expressed as a sum of an interaction plus independent regional Lagrangians. The resulting set of EL equations, written in terms of "average neurons," can be reduced to those used in other studies that consider the dipole-layered effects of long-range interactions [86,87]. However, this study permits inclusion of the short-range interactions.

Rigorous theorems exclude second order phase transitions in isotropic O(2) (rotationally invariant) 2-dimensional GL systems with numbers of field components ≥ 2, due to "spin-wave" fluctuations that compete with couplings between spins [44,78], considered to be firings in this study. Many rotationally equivalent long-ranged periodic configurations, waves, each with an otherwise nonzero average spin, interfere to give a net average of zero. However, the Lagrangian of these coupled M^G fields does not possess the O(2) algebraic structure assumed to describe this phenomena, permitting a discrete set of minima to exist that can likely withstand these particular fluctuations even under critical conditions.

Similarly, other arguments for "topological order" cannot be straightforwardly applied [9,58,59]. Still, this system may support some topological excitations, solitary waves. Most investigations have been done only in 1-dimensional systems [10], including some neuronal reaction-diffusion models [105]. A 1-dimensional excitation might propagate in 2 dimensions, with a contribution to the "rest mass" (b-coefficient of M^2) proportional to $\int dM\, (g_G g^G)^{\frac{1}{2}}$, but similar to the discussion above, these models typically possess symmetries not directly appropriate to the neocortex. An important, quite general benefit to be derived by considering these excitations is that dispersive gradient effects, localized defects, and boundary conditions are many times more easily analytically treated than by other formal methods by dealing directly with the EL equations [93,94].

5. Calculations

5.1. *Examples*

Straightforward calculations of the approximate potential surface $\bar{\Upsilon}^{(0)}$, using uniform $\bar{M}^G$ of Eq. (4.17), have been performed. Calculated is:

$$\overline{\underline{L}} \equiv \underline{L}^0(\bar{M}^G) = \tfrac{1}{2} N^{-1}[g_{EE}(g^E)^2 + g_{II}(g^I)^2]. \tag{5.1}$$

Some systematics are suggested from searching out minima of $\bar{\underline{L}}$ for a few sets of mesoscopic parameters.

Note that minima of $\bar{\underline{L}}$ are sought, not minima of $\bar{\underline{L}}_e$ derived from Eq. (4.20). The contribution to $\bar{\underline{L}}_e$ from $\ln(\tau g^{-\frac{1}{2}})$ is typically > 0, except for regions of very large$|F^G|$. For example, for $N^E = 125$ and $N^I = 25$, $\ln(\tau g^{-\frac{1}{2}}) \le 0$ for $|F^G| \ge 4.717$, not at all a typical value for reasonable neuronal parameters given in Eq. (3.3). The effects of this ln contribution, relative to $\bar{\underline{L}}$, are to give a positive translation and to globally warp the surface of $\bar{\underline{L}}$ over $\bar{M}^G$-space.

Example a. Consider the following rather arbitrary example of a region of mesocolumns, each with 150 neurons, selected to model minicolumns and to symmetrize the mesoscopic parameters, with $\gamma^G = 0$. Take $N^E = 125$, $N^I = 25$, $V^G = 10$ mV, $A^G = 1.5$, $B^G = .25$, $v^G = .1$ mV $= \phi^G$. This yields $\alpha^G = .005$, $\beta^G = 4.886$, $\gamma^G = 0$.

A search reveals a minimum at $\bar{M}^G = 0$, $\bar{\underline{L}} = 0$. At this minima, $F^G = g^G = 0$, $g_{EE} = .008$, $g_{II} = .04$. Also, $g^E{}_{;E} = g^E{}_{,E} = 2.054$, $g^I{}_{;I} = g^I{}_{,I} = -.3892$, $R = .179$, and $H^G = 0$. This surface has a least steep span across the axis maintaining $F^G = 0$, where $\not{M} = 0$. For example, $\bar{\underline{L}} = .1$ at $\bar{M}^G = 25$ and at $\bar{M}^G = -25$; $\bar{\underline{L}} = 1.539$ at $\bar{M}^E = \pm 25 = -M^I$. About this minimum, a small difference in $\bar{M}^G$ can change $\exp(-N\bar{\Upsilon}^{(0)})$ to a substantially lower probability of being in the altered firing state. For instance, $\bar{\underline{L}} = 3.929 \times 10^{-4}$ at $\bar{M}^E = 5$, $\bar{M}^I = 0$, gives the probability for this state as $\exp(-\Lambda u N \bar{\underline{L}}) \times$ the probability at the minimum, which is vanishingly small for $\Lambda u N \approx 10^{10}$.

The following notation will be used to represent calculations only at extrema (demonstrated by calculations to be minima): At $\ll \bar{M} \gg$, calculate $(\bar{M}^E, \bar{M}^I; \bar{\underline{L}})$, and the coefficients of $((\nabla M^E)^2 // (\nabla M^I)^2)$ and $((\dot{M}^E)^2 : (\dot{M}^I)^2 :: \dot{M}^E : \dot{M}^I)$ that contribute to $\underline{L}$ in Eq. (4.12). For the above $\ll \bar{M} \gg$, obtain (0, 0; 0), (0 // 0), $(2.67 \times 10^{-5} : 1.33 \times 10^{-4} :: 0 : 0)$. Note that the differential factors of these coefficients vary slowly and are of long wavelength in macroscopic regions, and are therefore also small.

-- Fig. 3 --

This minimum might seem to be a classic representation of a disordered state of firings. Fig. 3 exhibits the cortical surface of $\bar{\underline{L}}$ for these parameters [91]. However, at a scale at which gradient interactions are still small contributions, a fine structure yielding other local minima becomes apparent. At the nearest integral values of $\bar{M}^G$, data at these local minima are included in Table 1 together with data calculated at global minima. The existence of pairs of degenerate minima suffice to recommend further investigation into second order phase transitions and into properties of solitary waves that may be supported in this instance [93,94].

-- Table 1 --

Example b1. If the mesoscopic parameters are changed from the above, by changing the synaptic conductivities from $A^G = 1.5$ to $A^E = 1.25$ and $A^I = 1.75$, this yields $\alpha^E = 4.762 \times 10^{-3}$, $\alpha^I = 5.185 \times 10^{-3}$, $\beta^E = 4.570$, $\beta^I = 5.182$, $\gamma^E = 9.524 \times 10^{-2}$, $\gamma^I = -7.407 \times 10^{-2}$. Note that $\bar{M}^G = 0$ is no longer a minimum. (See Table 1.) Fig. 4 exhibits the cortical surface of $\bar{\underline{L}}$ for these parameters.

-- Fig. 4 --

Example b2. Example b1 is changed to $A^E = 1.75$, $A^I = 1.25$. (See Table 1.)

Example c. Example b1 is changed to $N^E = 150$, $N^I = 30$. This retains the same ratio of N^E/N^I, but increases N. (See Table 1.)

Example d. Example b1 is changed to $N^E = 150$, $N^I = 50$. This decreases the ratio N^E/N^I and increases N. (See Table 1.)

As calculated in Table 1, the gradient couplings cause NN mesocolumns to fire M^E (M^I) oppositely (similarly), in accord with empirical observations of interacting columnar structures that favor periodically alternating firings, supporting the development of Secs. 2 and 3. Of course, Sec. 4 is also essential to establish a variational principle, to demonstrate the stability of such structures in the presence of nonlinearities and (weak) fluctuations.

In the vicinity surrounding a minimum, the extrema equations can be fit to a GL polynomial, as permitted by Eq. (4.21). Some preliminary least-square fits have been performed, using mesocolumnar parameters within the ranges of the above Examples. Long-ranged J^G influences were also included, causing substantial shifts in $\ll \bar{M}^G \gg$ for J^G terms on the order of $\underline{L}$. These calculations confirm that linear terms in the EL equations are typically much larger than higher order terms, for quite large values of $M^G - \ll \bar{M}^G \gg$, and that extrema are quite sensitive to nonlinearities and to long-ranged afferents.

5.2. *Oscillatory Behavior*

An examination of the Lagrangian shows that the linearized EL equations for the firing deviations from the static (nonlinear) extrema can be written as:

$$0 = - \underline{f}_{|G|} \ddot{\underline{M}}^{|G|} + \underline{f}^1_G \dot{\underline{M}}^{G'} - \underline{g}_{|G|} \nabla^2 \underline{M}^{|G|} + \underline{b}_{|G|} \underline{M}^{|G|} + \underline{b}\,\underline{M}^{G'} \ , \tag{5.2}$$

$$\underline{M}^G = M^G - \ll \bar{M}^G \gg \ ,$$

$$\underline{f}^1_E = - \underline{f}^1_I \equiv \underline{f} \ ,$$

where the vertical bars indicate no sum is to be taken over repeated indices, and $G \neq G'$. The first step towards finding a weakly nonlinear oscillatory steady state [79] is to examine the spatially-temporally Fourier-transformed linearized EL equations. Define

$$\underline{M}^G_{osc}(z,t) = \int d^2\xi d\omega \ \hat{\underline{M}}^G_{osc}(\xi,\omega) \exp[i(\xi \cdot z - \omega t)], \tag{5.3}$$

and obtain a dispersion relation for $\omega(\xi)$:

$$2A(\omega\tau)^2 = - B \pm (B^2 - 4AC)^{\frac{1}{2}} \ , \tag{5.4}$$

$$A = \underline{f}_E \underline{f}_I > 0,$$

$$B = (\underline{f}_E \underline{b}_I + \underline{f}_I \underline{b}_E - \underline{f}^2) + (\xi\rho)^2 (\underline{f}_E \underline{g}_I + \underline{f}_I \underline{g}_E) \ ,$$

$$C = (\underline{b}_E \underline{b}_I - \underline{b}^2) + (\xi\rho)^2 (\underline{g}_E \underline{b}_I + \underline{g}_I \underline{b}_E) + (\xi\rho)^4 \underline{g}_E \underline{g}_I \ .$$

This condition also fixes the ratio $\underline{M}^E_{osc} / \underline{M}^I_{osc}$.

Empirical observations of macroscopic EEG activity, reasonably extrapolated to nondispersive flat z-surfaces [87], estimate $\omega < 10^2 \text{sec}^{-1}$ and $\xi < 10^{-1} \text{cm}^{-1}$. $\omega = 2\pi q \times$frequency and $\xi = 2\pi r/$wavelength, where q and r are integers representing normal modes. Since $\tau \approx 10^{-2}$ sec and $\rho < 10^{-2}$ cm, $\omega\tau < 1$ and $\xi\rho < 10^{-3}$. Therefore B, C, and ω^2 may be relatively independent of $(\xi\rho)^2$ for many values of mesoscopic parameters, consistent with the lack of spatial correlation of much of EEG data.

All the Examples in Table 1 yield $B > 0$ and $C > 0$. Therefore ω is imaginary and no oscillatory steady state exists about these static extrema. Rather, these Examples exhibit behavior typical of the "low temperature" side of phase transitions [69], reinforcing motivation to continue RG studies in this system.

Oscillatory steady states with $B > 0$ and $C < 0$ can be induced in the above Examples by the long-ranged J_G sources. For instance, if the static extremum of Example b2 is driven to $\bar{M}^G = 0$ by turning on $J_E = -1.3$ and $J_I = 1.5$, then $C/A \approx -2.1 - 6.3(\xi\rho)^2$ and $\frac{1}{2} B/A = 1.0 - .55(\xi\rho)^2$, whereas $C/A \approx 1.0 - 4.4(\xi\rho)^2$ and $\frac{1}{2} B/A = 1.0 - 3.5(\xi\rho)^2$ with $J_G = 0$. This is one example of the mechanism hypothesised in Sec. 4.6, wherein J_G afferents can induce phase transitions. This also stresses the importance of long-ranged interactions.

Oscillatory steady states can also arise with $B < 0$. Different coefficients in Eq. (5.2), and therefore different mesocolumnar mechanisms, are involved. Applying $J_E = -2.6$ and $J_I = 4.9$ to Example b2 produces an extremum shifted to $\bar{M}^E = 25$ and $\bar{M}^I = 5$, with $\frac{1}{2} B/A = -.72 - 6.7(\xi \rho)^2$. Note that all these spatially uncorrelated steady states are consistent with the empirical $\omega\tau < 1$.

It is also interesting to examine oscillatory steady states by yet other mechanisms that yield $C \approx O[(\xi \rho)^2]$. Since the $\underline{g}_G$-coefficients enter, this also tests the development of Secs. 2 and 3. Eq. (5.4) yields:

$$\omega = \hat{v} \xi \ , \tag{5.5}$$

were $\hat{v}$ is the wave propagation rate. In this oscillatory state, low-frequency long-wavelength propagation of information has minimal oscillatory background. $J_E = -.53$ and $J_I = -.11$ applied to Example b2 produces an extremum shifted to $\bar{M}^E = -3.0$ and $\bar{M}^I = 1.5$, with $\frac{1}{2} B/A = .74 - .24(\xi \rho)^2$ and $C/A = -2.5(\xi \rho)^2 + .025(\xi \rho)^4$. This yields $\hat{v} \approx 1$ cm/sec, consistent with the rate of information processing between mesocolumns.

In the above 3 examples of oscillatory states driven by long-ranged sources, the gradient couplings cause NN mesocolumns to fire M^G similarly. These examples support the hypothesis that low-frequency long-wavelength interactions between short-ranged neurons, modulated by long-ranged afferents, are primarily correlated with intra-regional information processing among mesocolumns. Since inter-regional interactions are facilitated at long-ranged excitatory action potential rates of 600 to 900 cm/sec [87], it is possible for several regions to simultaneously process similar information at the slower intra-regional rate.

6. Conclusion

The following assumptions and their derived conclusions have been formulated. These assumptions, although supported by arguments resting on the cited references, may be correctly viewed as gross simplifications of the complex microscopic details of neocortical function. However, the detailed analytic development of even these assumptions and their logical conclusions has not been previously formulated, and much empirical neuronal information is explicitly retained without adding any undefined or unphysical parameters.

Neocortical information is assumed to be statistically processed primarily by voltage-gated presynaptic and chemically gated postsynaptic interactions that take place on the molecular scale of membrane physics. Averages over tens of thousands of synapses are taken to derive the conditional probability for neuronal firing. Similar to other physical systems, an averaging over these microscopic degrees of freedom, whether they be deterministic or stochastic, gives rise to a stochastic macroscopic description.

Short-ranged short-time neuronal interactions are assumed to be primarily mediated by neuronal processes within nearest-neighbor distances of empirically observed minicolumns. Averages over hundreds of neurons, respecting the independence of excitatory and inhibitory transmitters, are taken to derive nearest-neighbor mesocolumnar gradient interactions and the conditional probability for regional activity of mesocolumnar firings. Similar to other physical systems, there exist local order parameters that describe the dynamics.

A Markovian development for the short-time conditional probability is assumed to describe the long-time response of a region. Long-time electrotonic and humoral influences, and long-ranged synaptic influences are consistently treated in this formulation. A temporal string of these conditional probabilities is developed to derive a Lagrangian formulation, expressing the statistical mechanics inherent in this system. Similar to other physical systems, the macroscopic nonequilibrium statistical mechanics can be described by an Onsager-Machlup type Lagrangian. Sufficient conditions for long wavelength, long period oscillatory behavior have been derived directly from the corresponding variational principle applied to the macroscopic statistics of interacting mesocolumns, driven by long-ranged extrinsic or inter-regional afferents. The spatial behavior of extrema is also consistent with observed columnar physiology.

Other features of this Lagrangian motivate further study into phase transitions.

As experienced in studies of other collective systems, the macroscopic activity of the neocortex is derived to depend only on the systematics of microscopic activity and on a few mesoscopic parameters, albeit some care must be taken in this development. This formulation also permits future approximation or elaboration within current paradigms of collective systems. For example, consistent with existing ranges of numerical values of all parameters, a Ginzburg-Landau expression is derived for neocortex.

Some tentative conjectures on biological mechanisms supported by this study are published elsewhere [48,49]. Calculations examining more complex spatial-temporal interactions typically investigated in other dissipative systems, and other more detailed computations on specific mechanisms will be presented in future papers.

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FIGURE CAPTIONS

FIG. 1. Illustrated are three biophysical scales of neocortical interactions: (a)-(a*)-(a’) microscopic neurons; (b)-(b’) mesocolumnar domains; (c)-(c’) macroscopic regions. In Fig (a*) synaptic inter-neuronal interactions, averaged over by mesocolumns, are phenomenologically described by the mean and variance of a distribution Ψ. Similarly, in Fig. (a) intra-neuronal transmissions are phenomenologically described by the mean and variance of Γ. (This is a modified freehand sketch from Ref. [100].) Mesocolumnar averaged excitatory (E) and inhibitory (I) neuronal firings, N and M, are represented in Fig. (a’). In Fig. (b) the vertical organization of minicolumns is sketched together with their horizontal stratification, yielding a physiological entity, the mesocolumn. In Fig. (b’) the overlap of interacting mesocolumns is sketched. In Fig. (c) macroscopic regions of neocortex are depicted as arising from many mesocolumnar domains. These are the regions designated for study here. Fig. (c’) sketches how regions may be coupled by long-ranged interactions.

FIG. 2. Represented is the relationship of this study to other neurological structures and processes. The macroscopic Lagrangian $\underline{L}$ in the triangle represents the interactions and fluctuations that process input to output patterns of mesocolumnar firings. These patterns, represented by the RHS and LHS circles, are expressed in terms of the probability eigenfunctions of $\underline{L}$. The b’s are the expansion coefficients of the polynomial representation of $\underline{L}$. The square, representing stored acquired/inherited patterns, may interact back on the triangle, changing the electrical and chemical parameters which affect the local and global information processing of $\underline{L}$.

FIG. 3. Illustrated is the surface of the Lagrangian $\overline{\underline{L}}$ over the $(\bar{M}^E - \bar{M}^I)$ plane, for Example a. Figs. (a), (b) and (c) are viewed from a distance of 10,000 grid units to minimize perceptual convergence, at an azimuth of -25 degrees, measured counterclockwise from the vector (0,0) to (0,-25), and an elevation of 30 degrees. The scale of $\overline{\underline{L}}$ is full in Fig. (a) at a height of $2.64 \times 10^3/\tau$. In Fig. (b), all points on the surface higher than $10^2/\tau$ are deleted, and in Fig. (c) the cutoff is at $5 \times 10^{-3}/\tau$. Another view of the fine structure at this latter cutoff is seen in Fig. (d), which is viewed from an azimuth of 135 degrees and an elevation of 45 degrees.

FIG. 4. Illustrated is the surface of $\overline{\underline{L}}$ for Example b1. The details of the viewing angles and scales are described in Fig. 3. Here, Fig. (a) is at full scale at a height of $3.06 \times 10^3/\tau$. Fig. (b) is cutoff at $5 \times 10^{-2}/\tau$, and Fig. (c) shows the same surface at the view described in Fig. 3(d).

TABLE CAPTIONS

TABLE 1. At minima $\ll \bar{M} \gg$, for the mesoscopic parameters given in the various Examples, calculated are $(\bar{M}^E, \bar{M}^I; \overline{\underline{L}})$, the coefficients of $((\nabla M^E)^2 // (\nabla M^I)^2)$ and $((\dot{M}^E)^2 : (\dot{M}^I)^2 :: \dot{M}^E : \dot{M}^I)$, and the values of the Riemannian contributions to the differential propagator, $\underline{V} - \underline{V}'$.